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Adaptive Output Feedback Control of Flow-Induced Vibrations of a Membrane at High Mach Numbers

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Abstract— **We address the problem of adaptive output-feedback stabilization for flow-induced vibrations of a membrane at high Mach numbers. The aeroelastic instability is compensated by using boundary control and anti-collocated measurement. The membrane is infinite in spanwise and the streamwise vibrations are modeled by a one-dimensional wave Partial Differential Equation (PDE) with an aerodynamic forcing term. Based on Piston theory, the term is represented by an anti-damping term multiplying a constant coefficient and a convective term multiplying an unknown constant coefficient. We then transform the wave PDE to a one-dimensional 2 × 2 first-order hyperbolic PDEs with constant coupling coefficients. Before introducing an adaptive output feedback controller for the hyperbolic PDEs, we present a nonadaptive explicit state feedback controller. To deal with the absence of both full-state measurements and parameter knowledge, the adaptive output-feedback design is used; the design is based on an observer canonical form which is obtained through a change of variables and a backstepping transformation. The form enables us to design an explicit state observer. We employ gradient-based parameter estimators. For the closed loop system, we achieve convergence of the state of the wave PDE to zero. We validate our result with a simulation.**

I. INTRODUCTION

Flow-induced structural vibrations occur in a variety of engineering fields, e.g., flows around aircraft wings, flows in a flexible pipe and flows around turbomachinery blades. Most of the aeroelastic phenomena involves the mutual interaction between inertial, elastic and aerodynamic forces, which can cause undesirable deformation, even failure of the structure. Aeroelastic instability problems at high Mach numbers have received a lot of attention [1]–[3].

Flow-induced vibration of an infinite membrane is a simplified model, where various control approaches have been tried. Both in-domain and boundary actuations (e.g. [4], [5]) have been used to stabilize the vibration of an elastic plate. The vibration of the infinite membrane is governed by a wave Partial Differential Equation(PDE) with aerodynamic forcing. Piston theory (see [6], [7]) is extensively used to model the aerodynamic pressure both in supersonic and hypersonic aeroelastic problems. Our work is based on a wave PDE model, studying the problem of flow-induced vibration of the infinite membrane with aerodynamic pressure defined by piston theory. The goal is to stabilize the wave PDE by applying a boundary control using an anti-collocated single measurement.

Boundary stabilization of unstable wave equations has been pursed in [8]–[11]. The wave PDE with in-domain anti-damping term is addressed in [8] and the output feedback stabilization of unstable wave PDE in [10]. For an unstable wave equation with an unmatched anti-damping term in boundary, adaptive boundary control is introduced in [12]. We address adaptive boundary control of wave PDEs with in-domain anti-damping term which has not been solved before.

We focus on the wave PDE with an in-domain anti-damping term coupled with a constant coefficient and a convective term with an unknown constant coefficient. The Control is applied at one boundary, whereas a measurement is taken from another. Both appear as Neumann boundary conditions. We design an adaptive output feedback controller for the system. Parameters of high Mach number flows are usually difficult to obtain in most wind tunnel experiments. Our work has the advantage to stabilize the wave PDE without the knowledge of flow parameters.

Through three different transformations, we obtain a 2×2 coupled first-order hyperbolic PDEs in observer canonical form in which unknown parameters multiply the measured output. By solving these, we obtain a parameter estimation model that is in the form of an integral equation relating delayed values of input and output. Based on this model, we estimate the parameters by employing a gradient-based method. Then we develop a filter-based adaptive observer and an output feedback controller that guarantees the convergence of state of the wave equation to zero. Finally, we illustrate our result with a simulation.

II. PROBLEM STATEMENT

A. Aeroelastic Model

We consider an aeroelastic problem at high Mach numbers, where we study the vibrations of membrane induced by aerodynamics forces. The membrane is infinite in the y direction with flow past above it in the x direction ($0 \le x \le 1$). Assuming that the membrane's vibrations are uniform in y, and varying along x direction, we model the displacement of the membrane in z direction by a one-dimensional wave equation with an aerodynamic forcing term,

$$w_{tt}(x,t) = c^2 w_{xx}(x,t) + p(x,t), \tag{1}$$

where w is the displacement of the membrane in z direction. It is assumed that constant tension forces are applied to the membrane and its density remains uniform in all directions. Therefore, the coefficient c is constant, relating the ratio of membrane's tension to linear density. The value $p(x,t)$

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represents the local aerodynamic pressure exerted by local fluid velocity normal to the membrane. Based on Piston theory, the pressure at high mach numbers is given by

$$p(x,t) = \frac{\rho V_\infty}{M}\left(\frac{\partial w(x,t)}{\partial t} + V_\infty \frac{\partial w(x,t)}{\partial x}\right), \tag{2}$$

where ρ is the free-stream fluid density, V_∞ is the free-stream fluid velocity, M is the Mach number. Substituting the pressure $p(x,t)$ in (1), we have

$$w_{tt} = c^2 w_{xx} + \mu w_t + \nu w_x, \tag{3}$$
$$w(0,t) = 0, \tag{4}$$
$$w_x(1,t) = U(t). \tag{5}$$

The control force U is applied at the trailing edge $(x=1)$ of the membrane while a measurement Y is taken at the fixed boundary $(x=0)$, so that $Y(t) = w_x(0,t)$. Considering that the membrane at $x=0$ is fixed to a wall, the displacement at $x=0$ equals to zero. $w_x(0,t)$ thus represents the angle of the slope between the x-axis and displacement of the membrane in z-axis, which can be measured by an inclinometer. The coefficients in (3)–(5) are given by

$$\mu = \rho a,\ \nu = \rho V_\infty a,\ a = \frac{V_\infty}{M}, \tag{6}$$

where a is the speed of sound. The value c, ρ, a are accessible form the knowledge of the system's properties, but the free-stream velocity V_∞ is unknown. Therefore, μ is known and ν is unknown.

B. Transformation to a 2×2 coupled first-order hyperbolic equations

We transform the wave equation to be two coupled first-order hyperbolic equations by changing variables. First, define $\bar{u} = cw_x - w_t$ and $\bar{v} = cw_x + w_t$, thus we have $w_x = \frac{\bar{u}+\bar{v}}{2c}$ and $w_t = \frac{\bar{v}-\bar{u}}{2}$. Substituting into (3), we obtain

$$\bar{u}_t = -c\bar{u}_x - \left(\frac{\nu}{2c} - \frac{\mu}{2}\right)\bar{u} - \left(\frac{\nu}{2c} + \frac{\mu}{2}\right)\bar{v}, \tag{7}$$
$$\bar{v}_t = c\bar{v}_x + \left(\frac{\nu}{2c} - \frac{\mu}{2}\right)\bar{u} + \left(\frac{\nu}{2c} + \frac{\mu}{2}\right)\bar{v}, \tag{8}$$

and the boundary conditions transform as $\bar{v}(1,t) = 2cw_x(1,t) - \bar{u}(1,t)$, $\bar{u}(0,t) = cw_x(0,t) - w_t(0,t)$ and $\bar{v}(0,t) = cw_x(0,t) + w_t(0,t)$. Since $w_t(0,t) = 0$, we arrive at

$$\bar{u}(0,t) = \bar{v}(0,t), \tag{9}$$
$$\bar{v}(1,t) = 2cU(t) - \bar{u}(1,t), \tag{10}$$

and $Y_0(t) = \bar{u}(0,t) = \bar{v}(0,t)$. Therefore we obtain a first-order hyperbolic 2×2 system with in-domain constant coupling coefficients.

III. Nonadaptive Explicit Full State Feedback Control

Assume that c, V_∞, ρ and a are known. Applying Theorem 1 in [13] to the coupled hyperbolic system (7)-(10), we obtain a full-state feedback control law

$$U(t) = \frac{1}{2c}\int_0^1 k_1(\xi)\bar{u}(\xi,t) + k_2(\xi)\bar{v}(\xi,t)d\xi + \frac{1}{2c}\bar{u}(1,t), \tag{11}$$

where k_1 and k_2 are control kernels. Plugging in the coefficients (6), the kernels are explicitly given by

$$k_1(\xi) = H(\xi)\left(F_0(\xi) + \sqrt{\frac{1-\xi}{1+\xi}}F_1(\xi) + F_\Pi(\xi)\right), \tag{12}$$
$$k_2(\xi) = H(\xi)\left(F_0(\xi) + \sqrt{\frac{1+\xi}{1-\xi}}F_1(\xi) + F_\Pi(\xi)\right), \tag{13}$$

where

$$H(\xi) = \frac{-1}{2c}\exp\left(\frac{-\rho V_\infty a}{2c^2}(1-\xi)\right), \tag{14}$$

$$F_0(\xi) = -\frac{\rho a}{2}\left(1+\frac{V_\infty}{c}\right)I_0\left(\frac{\rho a}{2}\sqrt{1-\left(\frac{V_\infty}{c}\right)^2}\sqrt{1-\xi^2}\right), \tag{15}$$

$$F_1(\xi) = \frac{\rho a}{2}\sqrt{1-\left(\frac{V_\infty}{c}\right)^2}\, I_1\left(\frac{\rho a}{2}\sqrt{1-\left(\frac{V_\infty}{c}\right)^2}\sqrt{1-\xi^2}\right), \tag{16}$$

$$F_\Pi = \frac{\rho V_\infty a}{2c}\Pi\left[\frac{\rho a}{4c}\left(\frac{V_\infty}{c}-1\right)(1-\xi), -\frac{\rho a}{4c}\left(\frac{V_\infty}{c}-1\right)(1+\xi)\right], \tag{17}$$

with I_n denoting the modified Bessel function of the first kind (of order n) and $\Pi(x,y) = e^{x+y}Q_1(\sqrt{2x}, \sqrt{2y})$, where Q_1 is the generalized Marcum Q-function of first order. Substituting $\bar{u}$ and $\bar{v}$ in (11), the full-state feedback controller for wave PDE is given by

$$U(t) = \frac{1}{c}\int_0^1 c(k_1(\xi)+k_2(\xi))w_t(\xi,t) + (k_1(\xi)-k_2(\xi))w_x(\xi,t)d\xi - \frac{1}{c}w_t(1,t). \tag{18}$$

The closed-loop system is exponentially stable in the L^2 sense. The controller in (11) needs full-state knowledge. To estimate both infinite-dimensional states, a dual method to design an explicit boundary observer is presented in [14]. We can obtain an explicit output feedback controller by combining the explicit boundary observer and explicit full-state feedback controller. Assume V_∞ is unknown, this design has difficulty extending to adaptive control since the relation between kernel variables and the unknown parameters are complex and nonlinear. Therefore, we present another way to design an adaptive output feedback control to deal with the unknown free-stream velocity.

IV. Adaptive Output Feedback Control

Assume c, ρ, a are known and V_∞ is unknown, thus ν is unknown and μ is known. To deal with the in-domain unknown cofficient multiplying with unknown state variable, we design an adaptive output feedback controller. Through two steps of transformation, we transform the system in (7)-(10) to an observer canonical form.

Define $\bar{c}_1 = \frac{\nu}{2c} - \frac{\mu}{2}$ and $\bar{c}_2 = \frac{\nu}{2c} + \frac{\mu}{2}$ and introduce scaled states (using an invertible transformation)

$$u(x,t) = \exp\left(\frac{\bar{c}_1}{c}x\right)\bar{u}(x,t),\, v(x,t) = \exp\left(\frac{\bar{c}_2}{c}x\right)\bar{v}(x,t), \tag{19}$$

which substituted in (7)-(10) give

$$u_t(x,t) = -cu_x(x,t) + c_1(x)v(x,t), \tag{20}$$
$$v_t(x,t) = cv_x(x,t) + c_2(x)u(x,t), \tag{21}$$
$$u(0,t) = v(0,t), \tag{22}$$
$$v(1,t) = \rho_0 U(t) + \rho_1 u(1,t), \tag{23}$$

where

$$c_1(x) = -\left(\frac{\nu}{2c} + \frac{\mu}{2}\right)\exp\left(\frac{-\mu}{c}x\right), \tag{24}$$
$$c_2(x) = \left(\frac{\nu}{2c} - \frac{\mu}{2}\right)\exp\left(\frac{\mu}{c}x\right), \tag{25}$$
$$\rho_0 = 2c\exp\left(\frac{\nu}{2c^2} + \frac{\mu}{2c}\right), \rho_1 = -\exp\left(\frac{\mu}{c}\right). \tag{26}$$

By changing variables, the constant paramaters μ and ν in (3) appear as unknown spatially varing parameters $c_1(x)$ and $c_2(x)$ in (21) and (22). The boundary parameter ρ_0 is unknown and ρ_1 is known. We define the output

$$Y_0(t) = u(0,t) = cw_x(0,t) = cY(t), \tag{27}$$

as a new measurement. By desigining an output feedback controller $U(t)$ for the coupled (u,v) system, the original wave equation is stabilized.

A. Observer canonical form

We start by using the following pre-transformation to transform the system into an observer canonical form,

$$\alpha(x,t) = u(x,t) - \int_0^x K_{11}(x,\xi)u(\xi,t)d\xi - \int_0^x K_{12}(x,\xi)v(\xi,t)d\xi, \tag{28}$$
$$\beta(x,t) = v(x,t) - \int_0^x K_{21}(x,\xi)u(\xi,t)d\xi - \int_0^x K_{22}(x,\xi)v(\xi,t)d\xi, \tag{29}$$

where the kernels $K_{ij}(x,\xi)(\forall i,j=1,2)$ evolve in the triangular domain $T = \{(x,\xi) : 0 \leq \xi \leq x \leq 1\}$. The observer canonical form is given by

$$\alpha_t = -c\alpha_x + \theta_1(x)Y_0(t), \tag{30}$$
$$\beta_t = c\beta_x + \theta_2(x)Y_0(t), \tag{31}$$

with boundary conditions

$$\alpha(0,t) = \beta(0,t), \tag{32}$$
$$\beta(1,t) = \rho_0 U(t) + \rho_1 \alpha(1,t) \triangleq \hat{U}(t), \tag{33}$$

and the output is given by

$$Y_0(t) = \alpha(0,t) = \beta(0,t). \tag{34}$$

The well-posedness of the kernel equations is proved in the Appendix of [14]. The values of the θ's are given by

$$\theta_1(x) = -cK_{11}(x,0) + cK_{12}(x,0), \tag{35}$$
$$\theta_2(x) = -cK_{21}(x,0) + cK_{22}(x,0). \tag{36}$$

Remark 1. *There exist known constants M_1 and M_2 such that for all $x \in [0,1]$, $|\theta_1(x)| \leq M_1$, $|\theta_2(x)| \leq M_2$, and a known constant L_0 such that $|\rho_0| \leq L_0$.*

B. Adaptive output-feedback control design

The adaptive control law is given by

$$\begin{aligned} U(t) = & \int_{t-\frac{1}{c}}^{t} \hat{K}_2(t-\xi)U(\xi)d\xi \\ & + \gamma\Bigg(\int_{t-\frac{2}{c}}^{t-\frac{1}{c}} \hat{K}_2(t-\xi-1)Y(\xi)d\xi \\ & + \int_{t-\frac{1}{c}}^{t} \hat{K}_2(ct-c\xi)\int_{\xi-\frac{1}{c}}^{\xi} \hat{\theta}_1(1-c\xi+c\mu)Y(\mu)d\mu d\xi \\ & + \int_{t-\frac{1}{c}}^{t}\int_{ct-c\xi}^{1} \hat{K}_2(\mu)\hat{\theta}_2(ct-c\xi+1-\mu)d\mu\frac{Y(\mu)}{\rho_1}d\xi \\ & - Y\left(t-\frac{1}{c}\right) - \int_{t-\frac{1}{c}}^{t} \hat{\theta}_1(1-ct+c\xi)Y(\mu)ds\Bigg), \end{aligned} \tag{37}$$

where $\gamma = \frac{c\rho_1}{\hat{\rho}_0}$. $\hat{\theta}_1(x,t)$, $\hat{\theta}_2(x,t)$ and $\hat{\rho}_0(t)$ are generated from the update laws. We obtain $\hat{K}_2(x,t)$ by solving online the following Volterra equation

$$\hat{K}_2(x,t) = -\hat{\theta}_2(x,t) + \frac{1}{c}\int_0^x \hat{K}_2(x-\xi,t)\hat{\theta}_2(\xi,t)d\xi. \tag{38}$$

To derive the update laws for adaptive estimates of parameters $\hat{\theta}_1(x,t)$, $\hat{\theta}_2(x,t)$ and $\hat{\rho}_0(t)$. we use the system's input/output map for the target system as a parametric model. We can easily find the input/output relation by solving system equations in (30) and (31) directly. The input/output model is given:

$$\begin{aligned} Y_0(t) = & \rho_1 Y_0\left(t-\frac{2}{c}\right) + \rho_1\int_{t-\frac{2}{c}}^{t-\frac{1}{c}} \theta_1(2-c(s-t))Y_0(s)ds \\ & + \int_{t-\frac{1}{c}}^{t} \theta_2(c(t-s))Y_0(s)ds + \rho_0 U\left(t-\frac{1}{c}\right) + \varepsilon(t), \end{aligned} \tag{39}$$

where $\varepsilon(t)$ is defined as the error of the parametric model. The value of $\varepsilon(t)$ is arbitrary for $t \in [0, \frac{2}{c}]$, depending on the initial values of α, β and $\varepsilon(t) = 0$ for $t \in [\frac{2}{c}, \infty)$.

We represent the input $\hat{U}(t)$ and the output $Y_0(t)$ with the transport PDEs. The signal $\alpha(1,t)$ is given by solving (30),

$$\alpha(1,t) = Y_0\left(t-\frac{1}{c}\right) + \int_{t-\frac{1}{c}}^{t} \theta_1(1-c(t-s))Y_0(s)ds. \tag{40}$$

We have $U(t) = \frac{1}{\rho_0}\left(\hat{U}(t) - \rho_1\alpha(1,t)\right)$ and the signal $\alpha(1,t)$ given in (40). We represent the input $\hat{U}(t)$, $U(t)$, the output $Y_0(t)$ and the delayed output $Y_0(t-\frac{1}{c})$ with the following transport PDEs:

$$\psi_t(x,t)=c\psi_x(x,t), \tag{41}$$
$$\psi(1,t)=\hat{U}(t), \psi(x,0)=\psi_0(x), \tag{42}$$
$$\chi_t(x,t)=c\chi_x(x,t), \tag{43}$$
$$\chi(1,t)=U(t), \chi(x,0)=\chi_0(x), \tag{44}$$
$$\phi_t(x,t)=-c\phi_x(x,t), \tag{45}$$
$$\phi(0,t)=Y_0(t), \phi(x,0)=\phi_0(x), \tag{46}$$
$$\omega_t(x,t)=-c\omega_x(x,t), \tag{47}$$
$$\omega(0,t)=Y_0\left(t-\frac{1}{c}\right), \omega(x,0)=\omega_0(x), \tag{48}$$

where $x\in[0,1]$, and ψ_0,ϕ_0,ω_0 are arbitrary initial conditions verifying boundary conditions. The explicit solutions to the PDE filters for $x\in[0,1], t>\frac{1}{c}$ are given by

$$\psi(x,t)=\hat{U}\left(t+\frac{x-1}{c}\right), \tag{49}$$
$$\chi(x,t)=U\left(t+\frac{x-1}{c}\right), \tag{50}$$
$$\phi(x,t)=Y_0\left(t-\frac{x}{c}\right), \tag{51}$$
$$\omega(x,t)=Y_0\left(t-\frac{x+1}{c}\right). \tag{52}$$

We use system's input/output model in (39) as a parametic model to estimate unknown parameters and use Remark 1 to limit the adaptive estimates $\hat{\theta}_1(x)$ and $\hat{\theta}_2(x)$ with projection. This projection guarantees pointwise boundedness of $\hat{\theta}_1(x)$, $\hat{\theta}_2(x)$ and $\hat{K}_2(x)$.

The update laws are based on the gradient algorithm with normalization and projection,

$$\partial_t\hat{\theta}_1(x)=\gamma_1(x)\frac{\mathrm{Proj}\left(\frac{\rho_1}{c}Y_0(t-\frac{2-x}{c})\tilde{\beta}(0,t),\hat{\theta}_1(x,t)\right)}{1+\int_{t-\frac{2}{c}}^{t}Y_0^2(\tau)d\tau+U^2(t-\frac{1}{c})}, \tag{53}$$
$$\partial_t\hat{\theta}_2(x)=\gamma_2(x)\frac{\mathrm{Proj}\left(\frac{1}{c}Y_0(t-\frac{x}{c})\tilde{\beta}(0,t),\hat{\theta}_2(x,t)\right)}{1+\int_{t-\frac{2}{c}}^{t}Y_0^2(\tau)d\tau+U^2(t-\frac{1}{c})}, \tag{54}$$
$$\partial_t\hat{\rho}_0=\gamma_3\frac{U(t-\frac{1}{c})\tilde{\beta}(0,t)}{1+\int_{t-\frac{2}{c}}^{t}Y_0^2(\tau)d\tau+U^2(t-\frac{1}{c})}, \tag{55}$$

where $\gamma_1(x)$, $\gamma_2(x)$ and γ_3 are positive adaptation gains. $Y_0(t-\frac{2-x}{c})$, $Y_0(t-\frac{1-x}{c})$ and $U(t-\frac{1}{c})$ are the regressors. According to the parametric model, adaptive estimation error $\tilde{\beta}(0,t)$ is given by

$$\begin{aligned}\tilde{\beta}(0,t)=&Y_0(t)-\rho_1Y_0\left(t-\frac{2}{c}\right)-\int_{t-\frac{1}{c}}^{t}\hat{\theta}_2\left(c(t-s)\right)Y_0(s)ds\\&-\rho_1\int_{t-\frac{2}{c}}^{t-\frac{1}{c}}\hat{\theta}_1\left(2-c(s-t)\right)Y_0(s)ds-\hat{\rho}_0U\left(t-\frac{1}{c}\right),\end{aligned} \tag{56}$$

Denote $\delta(t)=1+\frac{1}{c^2}||\omega(t)||^2+\frac{1}{c^2}||\phi(t)||^2+\chi(0)^2$. With the filters, we rewrite the update laws as

$$\partial_t\hat{\theta}_1(x)=\mathrm{Proj}\left(\tau_1(x,t),\hat{\theta}_1(x,t)\right), \tag{57}$$
$$\partial_t\hat{\theta}_2(x)=\mathrm{Proj}\left(\tau_2(x,t),\hat{\theta}_2(x,t)\right), \tag{58}$$
$$\partial_t\hat{\rho}_0=\gamma_3\frac{\chi(0,t)\tilde{\beta}(0,t)}{\delta(t)}, \tag{59}$$

where

$$\tau_1(x,t)=\frac{\gamma_1(x)\rho_1}{c\delta(t)}\omega(1-x,t)\tilde{\beta}(0,t), \tag{60}$$
$$\tau_2(x,t)=\frac{\gamma_2(x)}{c\delta(t)}\phi(x,t)\tilde{\beta}(0,t). \tag{61}$$

The projection operator is given by

$$\mathrm{Proj}(\tau_i,\hat{\theta}_i)=\begin{cases}\tau_i, & |\hat{\theta}_i|<M_i \quad \text{or} \quad \hat{\theta}_i\tau_i\le 0,\\ 0, & |\hat{\theta}_i|=M_i \quad \text{and} \quad \hat{\theta}_i\tau_i>0,\end{cases} \tag{62}$$

and guarantees the pointwise boundedness of $\hat{\theta}_i(x)$. Denote the parameter estimation error as $\tilde{\theta}_i(x,t)=\theta_i(x)-\hat{\theta}_i(x,t), i=1,2$, and $\tilde{\rho}_0(t)=\rho_0-\hat{\rho}_0(t)$.

Lemma 1. *The adaptive laws* (57)-(59) *guarantee that:*

$$|\hat{\theta}_1(x)|\le M_1, |\hat{\theta}_2(x)|\le M_2, \tag{63}$$
$$||\tilde{\theta}_1||, ||\tilde{\theta}_2||, \tilde{\rho}_0\in\mathscr{L}_\infty, \tag{64}$$
$$||\partial_t\hat{\theta}_1||, ||\partial_t\hat{\theta}_2||, \partial_t\rho_0, \frac{\tilde{\beta}(0,t)}{\sqrt{\delta(t)}}\in\mathscr{L}_2\cap\mathscr{L}_\infty. \tag{65}$$

Theorem 1. *Consider the plant* (3)-(6) *under Remark 1 with the adaptive controller* (37) *and update laws* (57)-(59). *For any initial conditions* $\hat{\theta}_1(\cdot,0),\hat{\theta}_2(\cdot,0)\in C^1[0,1]$, $\hat{\rho}_0\in\mathbb{R}$, $u_0,v_0,\phi_0,\hat{\psi}_0,\chi_0,\omega_0$ *verifying the boundary conditions, the solution* $(u,v,\phi,\hat{\psi},\chi,\omega,\hat{\theta}_1,\hat{\theta}_2,\hat{\rho}_0)$ *is bounded for* $t\ge 0$ *and*

$$\lim_{t\to\infty}u(x,t)=0, \tag{66}$$
$$\lim_{t\to\infty}v(x,t)=0, \tag{67}$$

implying

$$\lim_{t\to\infty}w(x,t)=0. \tag{68}$$

The proof is derived as follows. First we use input and output filters to construct explicit adaptive observers. Then by the backstepping transformation, we transform adaptive state estimates to a cascade system, in which we derive the adaptive controller and prove the closed-loop stability of the system. After establishing the pointwise boundedness of state variables and filters, we obtain the convergence of (u,v)-system and therefore of the vibrations of the membrane.

We introduce the adaptive state estimates,

$$\hat{\alpha}(x,t)=\phi(x,t)+\frac{1}{c}\int_0^x\hat{\theta}_1(\xi)\phi(x-\xi,t)d\xi, \tag{69}$$
$$\hat{\beta}(x,t)=\hat{\psi}(x,t)+\frac{1}{c}\int_x^1\hat{\theta}_2(\xi)\phi(\xi-x,t)d\xi. \tag{70}$$

Now the input filter is defined for $\bar{U}(t)=\hat{\rho}_0U(t)+\rho_1\hat{\alpha}(1,t)$. The signal $\hat{\alpha}(1,t)$ is given by (40) with updated $\hat{\theta}_1$.

$$\hat{\psi}_t(x,t)=c\hat{\psi}_x(x,t), \tag{71}$$
$$\hat{\psi}(1,t)=\bar{U}(t), \tag{72}$$
$$\hat{\psi}(x,0)=\hat{\psi}_0(x). \tag{73}$$

These estimates verify

$$\hat{\alpha}_t = -c\hat{\alpha}_x + \hat{\theta}_1(x)Y_0(t) + \frac{1}{c}\int_0^x \partial_t\hat{\theta}_1(\xi)\phi(x-\xi,t)d\xi, \quad (74)$$

$$\hat{\beta}_t = c\hat{\beta}_x + \hat{\theta}_2(x)Y_0(t) + \frac{1}{c}\int_x^1 \partial_t\hat{\theta}_2(\xi)\phi(\xi-x,t)d\xi, \quad (75)$$

with boundary conditions

$$\hat{\alpha}(0,t) = \phi(0,t) = Y_0(t) = \alpha(0,t), \quad (76)$$

$$\hat{\beta}(1,t) = \hat{\psi}(1,t) = \bar{U}(t). \quad (77)$$

Denote the adaptive observer errors

$$\tilde{\alpha} = \alpha - \hat{\alpha}, \quad (78)$$

$$\tilde{\beta} = \beta - \hat{\beta}, \quad (79)$$

which lead to the error system

$$\tilde{\alpha}_t = -c\tilde{\alpha}_x + \tilde{\theta}_1(x)Y_0(t) - \frac{1}{c}\int_0^x \partial_t\hat{\theta}_1(\xi)\phi(x-\xi,t)d\xi, \quad (80)$$

$$\tilde{\beta}_t = c\tilde{\beta}_x + \tilde{\theta}_2(x)Y_0(t) - \frac{1}{c}\int_x^1 \partial_t\hat{\theta}_2(\xi)\phi(\xi-x,t)d\xi, \quad (81)$$

with boundary conditions

$$\tilde{\alpha}(0,t) = 0, \quad (82)$$

$$\tilde{\beta}(1,t) = \bar{U}(t) - \hat{U}(t) = \rho_1\tilde{\alpha}(1,t) + \tilde{\rho}_0 U(t). \quad (83)$$

We apply a backstepping transformation to the adaptive state estimates and obtain the target system given by

$$\zeta(x) = \hat{\alpha}(x) - \frac{1}{c}\int_0^x \hat{K}_1(x-\xi)\hat{\alpha}(\xi)d\xi := \mathscr{T}_1[\hat{\alpha}](x), \quad (84)$$

$$\eta(x) = \hat{\beta}(x) - \frac{1}{c}\int_0^x \hat{K}_2(x-\xi)\hat{\beta}(\xi)d\xi := \mathscr{T}_2[\hat{\beta}](x), \quad (85)$$

where $\hat{K}_1$ and $\hat{K}_2$ are obtained by the Volterra equations.

$$\hat{K}_1(x,t) = \hat{\theta}_1(x,t) - \frac{1}{c}\int_0^x \hat{K}_1(x-\xi,t)\hat{\theta}_1(\xi,t)d\xi, \quad (86)$$

$$\hat{K}_2(x,t) = -\hat{\theta}_2(x,t) + \frac{1}{c}\int_0^x \hat{K}_2(x-\xi,t)\hat{\theta}_2(\xi,t)d\xi. \quad (87)$$

The inverse transformations are given by

$$\hat{\alpha} = \zeta(x) - \frac{1}{c}\int_0^x \hat{\theta}_1(x-\xi)\hat{\alpha}(\xi)d\xi := \hat{\zeta} - \frac{1}{c}\hat{\theta}_1 * \hat{\zeta}, \quad (88)$$

$$\hat{\beta} = \eta(x) - \frac{1}{c}\int_0^x \hat{\theta}_2(x-\xi)\hat{\beta}(\xi)d\xi := \hat{\eta} - \frac{1}{c}\hat{\theta}_2 * \hat{\eta}. \quad (89)$$

With a lengthy but straightforward calculation, we obtain $\zeta = \phi$. The target system is given by

$$\begin{aligned}\eta_t =& c\eta_x - \hat{K}_2(x)\tilde{\beta}(0) + \eta * \mathscr{T}_2[\partial_t\hat{\theta}_2](x) \\ &+ \frac{1}{c}\mathscr{T}_2\Big[\int_x^1 \partial_t\hat{\theta}_2(\xi)\phi(\xi-x,t)d\xi\Big], \end{aligned}\quad (90)$$

$$\eta(1) = 0, \quad (91)$$

$$\phi_t = -c\phi_x, \quad (92)$$

$$\phi(0) = Y_0(t) = \eta(0) + \tilde{\beta}(0). \quad (93)$$

Thus we obtain the stability of original system $(u, v, \hat{\psi}, \phi)$ by studying the system in the equivalent variables $(\tilde{\alpha}, \tilde{\beta}, \phi, \eta)$. The variables $(\tilde{\alpha}, \tilde{\beta}, \phi, \eta)$ are governed by the following cascade PDEs system.

$$\tilde{\alpha}_t = -c\tilde{\alpha}_x + \tilde{\theta}_1(x)Y_0(t) - \frac{1}{c}\int_0^x \partial_t\hat{\theta}_1(\xi)\phi(x-\xi,t)d\xi, \quad (94)$$

$$\tilde{\alpha}(0,t) = 0, \quad (95)$$

$$\tilde{\beta}_t = c\tilde{\beta}_x + \tilde{\theta}_2(x)Y_0(t) - \frac{1}{c}\int_x^1 \partial_t\hat{\theta}_2(\xi)\phi(\xi-x,t)d\xi, \quad (96)$$

$$\tilde{\beta}(1,t) = \rho_1\tilde{\alpha}(1,t) + \tilde{\rho}_0 U(t). \quad (97)$$

$$\begin{aligned}\eta_t =& c\eta_x - \hat{K}_2(x)\tilde{\beta}(0) + \eta * \mathscr{T}_2[\partial_t\hat{\theta}_2](x) \\ &+ \frac{1}{c}\mathscr{T}_2\Big[\int_x^1 \partial_t\hat{\theta}_2(\xi)\phi(\xi-x,t)d\xi\Big], \end{aligned}\quad (98)$$

$$\eta(1) = 0, \quad (99)$$

$$\phi_t = -c\phi_x, \quad (100)$$

$$\phi(0) = Y_0(t) = \eta(0) + \tilde{\beta}(0). \quad (101)$$

The η-system is driven by the $\tilde{\beta}$, the ϕ-system is driven by the η-system and the $\tilde{\beta}$ system. The $\tilde{\beta}$ system is driven by $\tilde{\alpha}$-system. By following the same steps as in [15], we prove the L_2 and pointwise boundedness of the $(\tilde{\alpha}, \tilde{\beta}, \phi, \eta)$ and thus derive the pointwise convergence to zero of (u, v). By the inverse backstepping transformation and chang of variables, we obtain pointwise convergence to zero of $w(x,t)$ and thus achieve adaptive output feedback stabilization of the vibrations of the membrane.

According to the backstepping transformation of $\hat{\beta}$ in (85), we obtain from (99) that

$$\bar{U}(t) = \int_0^1 \hat{K}_2(1-\xi)\hat{\beta}(\xi,t)d\xi. \quad (102)$$

Substituting $\bar{U}(t) = \hat{\rho}_0 U(t) + \rho_1\hat{\alpha}(1,t)$, we have

$$U(t) = \frac{1}{\hat{\rho}_0}\int_0^1 \hat{K}_2(1-\xi)\hat{\beta}(\xi,t)d\xi - \frac{\rho_1}{\hat{\rho}_0}\hat{\alpha}(1,t). \quad (103)$$

Substituting $\hat{\beta}(x,t)$, $\hat{\alpha}(1,t)$ and $Y_0(t)$ into (103), the explicit adaptive controller is given by (37).

V. Simulations

We present the results of numerical simulation for the wave equation (3). The parameters are $c = 1$, $\mu = 0.1$ and $\nu = 1$. We see from Fig.1 that the open-loop is unstable and oscillatory; the closed-loop system in Fig. 2 describes that the adaptive control ensures the convergence of $w(x,t)$ to zero for all $x \in [0,1]$. The Fig.4 shows that the value of $\hat{\rho}_0$ converge to 3.466. Substituting it in (26), the estimate value of ν agrees with its given value. A higher gain function in update laws will induce a faster convergence of state $w(x,t)$.

VI. Conclusions

In this paper we solve the problem of adaptive output feedback stabilization for flow-induced vibrations of a membrane at high Mach numbers. The aeroealstic model we deal with is a wave PDE with an in-domain anti-damping term coupled with a constant coefficient and a convective

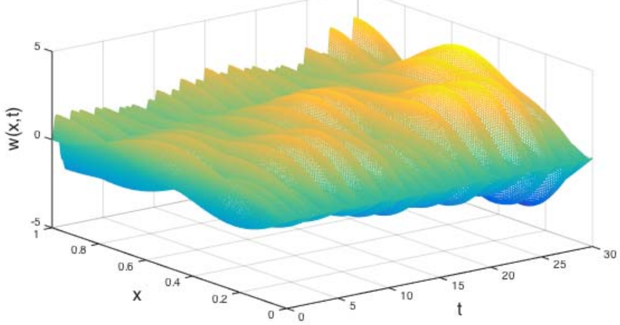

Fig. 1: Evolution of state $w(x,t)$ when $U(t) = 0$.

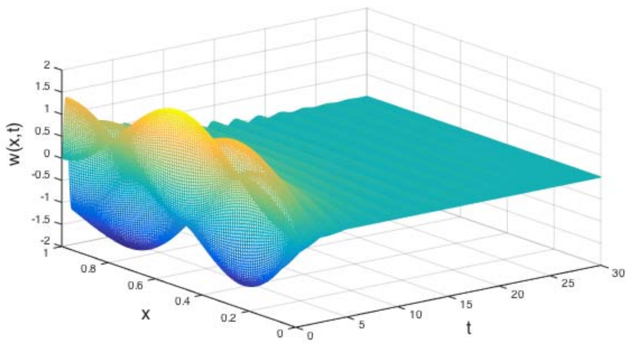

Fig. 2: Response of (3)-(6) to the adaptive controller (37).

term with an unknown constant coefficient. We achieve the pointwise convergence to zero of the state, whereas the flow velocity is estimated by the adaptive update laws, as shown in simulations. For future research, it is of interest to explore the wave PDE with unknown parameter coupling the anti-damping term. More control efforts are required since anti-damping term is the main cause of the instability. The simulation in this paper is to demonstrate the adaptive output stabilization of the aeroealstic model. The simulation that the values of flow coefficients are chosen from empirical data will be the subject of future work.

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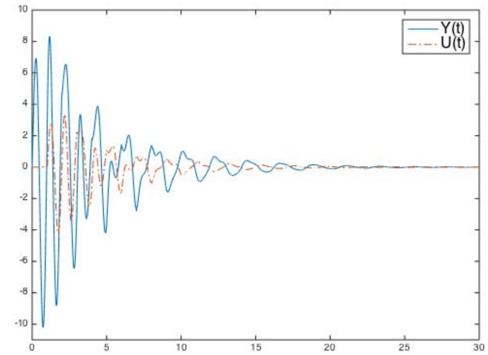

Fig. 3: Input and output of the closed-loop system.

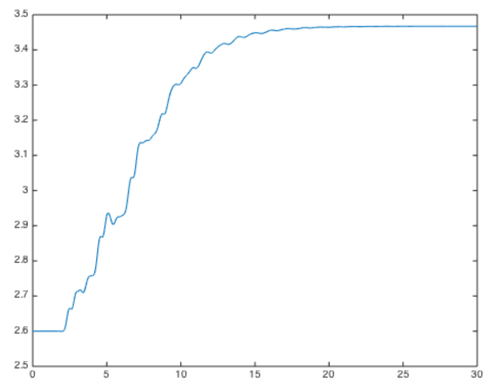

Fig. 4: Evolution of $\hat{\rho}_0(t)$.

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