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Safety-critical Traffic Control for Mixed Autonomy Systems with Input Delay and Disturbances

Chenguang Zhao, Huan Yu*

Abstract— **While the connected automated vehicle (CAV) has been applied in vehicle-based traffic control that aims to stabilize a string of car-following human-driven vehicles (HV), the safety impact of a CAV controller on the overall traffic flow system is still an open question. In this paper, we propose a Robust Safety-critical Traffic Control (RSTC) design to impart safety for the mixed autonomy traffic system, in which the speed disturbance from a leading HV and input delay of the stabilizing CAV controller are considered. We employ Control Barrier Function (CBF) to impose safety constraints on a nominal CAV control input that achieves string stability. The key challenge lies in incorporating effect of the input delay and external disturbances into the CBF constraints. The predicted speed and spacing gap in the robust CBF design is obtained using a delay-compensating state predictor. The forward invariance of the safe set is proved given the derivative of speed disturbance, i.e., acceleration of the leading vehicle, is bounded. The safety improvement of RSTC over the nominal controller is then validated via numerical simulations.**

Index Terms— **Connected and automated vehicle, String stability, Input delay, Disturbance, Control barrier function**

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I. Introduction

Connected automated vehicles (CAVs) have shown great potential for improving traffic. Mix-autonomy traffic, in which the CAV coexists with human-driven vehicles (HVs), has received considerable attention. This is because a long transition period is expected before fully automated traffic becomes a reality. In particular, a number of recent studies [19] have focused on stabilizing mixed autonomy traffic and reducing stop-and-go waves with control strategies designed for CAV. But research on their safety effect is still insufficient. In fact, a stabilizing-traffic controller can cause rear-end collisions in some scenarios [7]. Moreover, safety is a major factor affecting the public acceptance of autonomous driving, particularly regarding the possibility of rear-end collisions [6]. Therefore, imparting safety on the CAV-controlled traffic systems in an uncertain environment is our main focus in this paper.

In the study of control systems under state constraints, control barrier function defines safety as the forward invariance of a given safe set, i.e., the system state always stays in the safe set [3]. In comparison with model predictive control (MPC) and classical optimal control that incorporate safety constraints and may incur heavy computational cost, control barrier function (CBF) [3], [12], [14] can directly synthesize a safety-critical controller from a wide selection of existing unsafe nominal controllers. Due to its flexibility, the CBF has been applied to multiple domains, such as bipedal robotic walking [10], multi-robot systems [5], and quadrotors [26].

Although there have been various formulations of stabilizing traffic controllers that focused on improving overall traffic performance [13], only a few attempts have been made in the application of CBF for traffic safety [18], [25], [27]. Among them, previous work of the author [27] proposed safety-critical traffic control (STC) framework that guarantees safety and stability of longitudinal dynamics of mixed autonomy traffic. The aforementioned studies assume that control inputs are implemented by acceleration and deceleration actions of CAV without delay. In practice, however, CAV is subject to various sources of input delay, including communication delay, processing delay, and powertrain delay [22]. The safety-critical design is significantly impacted by the input delay of CAV, during which the safety constraints could be violated.

For the controlled systems with input delay, predictor feedback design is usually employed to eliminate the effect of delay by predicting the future state over the delay interval given the current and historical dynamics [1], [17]. Such a predictor is integrated into safety-critical control by using the predicted state to construct the CBF constraint and solve the current input value. One main challenge of applying predictor-based safety-critical control in traffic is the existence of external disturbances that evolve independently from the control input, such as the speed of a leading vehicle ahead of the controlled CAV. Future value of the speed disturbance is unavailable, and only inaccurate estimation can be used to design the controller. The naive combination of CBF and inaccurate disturbance will fail to maintain safety of the system [16].

Several recent efforts have been made for safety-critical control under disturbances. The notion of input-to-state safety is introduced in [2], [11], [15], which allows bounded safety violation by ensuring the forward invariance of a larger safe set than the original one without disturbances. In [8], [16], the problem is formulated as a second-order cone program with a non-smooth constraint, which precludes existing methods for closed-form solutions.

To summarize, safety-critical control with input delay and external disturbances has yet to be addressed. Motivated with mixed-autonomy traffic control systems, the contribution of this paper is both theoretical and applciation relevant. We design a robust STC (RSTC) controller for traffic systems under input delay and disturbances. The CBF-based RSTC

Chenguang Zhao and Huan Yu are with the Hong Kong University of Science and Technology (Guangzhou), Thrust of Intelligent Transportation, Guangzhou, Guangdong, China. Huan Yu is also affiliated with the Hong Kong University of Science and Technology, Department of Civil and Environmental Engineering, Hong Kong SAR, China.
* corresponding author (huanyu@ust.hk).

is able to guarantee safety of CAV controlled traffic systems in an uncertain and practical environment. Specifically, we construct and validate the RSTC from the following aspects:

- To compensate for the input delay, we design a state predictor and prove the upper bound of prediction error when there is an external disturbance from unpredictable leading vehicle speed. To guarantee safety with the inaccurate prediction and disturbance, we design robust CBF candidates with relative degree one for both CAV and HVs. A Quadratic Programming (QP) is formulated to synthesize a safe controller from a nominal controller that achieves string stability.
- We prove the forward invariance of the safe set for CAV and HV in Theorem 2 under the assumption of a bounded derivative of the disturbance, which indicates the bounded value of acceleration in practice. We also run simulations to analyze the safety improvement of the proposed RSTC in two safety-critical scenarios that can cause rear-end collisions in real traffic.

The rest of this paper is organized as follows. Section II formulates the mix-autonomy traffic model and introduces a nominal feedback controller. Section III designs a state predictor and then proposes CBF constraints to guarantee safety under disturbances. Section IV provides numerical simulation and safety improvement is validated.

II. Nominal Control of Mix autonomy traffic model

In this section, we formulate the microscopic car-following model to describe the longitudinal dynamics of mixed-autonomy traffic. Then we give a nominal feedback controller that achieves string stability for the system without delay.

A. *Mix-autonomy car-following traffic model*

We employ a general formulation of car-following models (e.g. intelligent driver's model, optimal velocity model [20]) to describe longitudinal dynamics of mix-autonomy traffic in which there are one leading CAV and N following HVs as shown in Fig. 1. The CAV follows a Head HV (HHV) and leads the HVs. For a HV i, its gap s_i and speed v_i, are governed by

$$\dot{s}_i(t) = v_{i-1}(t) - v_i(t), \tag{1}$$
$$\dot{v}_i(t) = F_i\left(s_i(t), \dot{s}_i(t), v_i(t)\right), \tag{2}$$

where v_{i-1} is the speed of its leading vehicle $i-1$, and F_i is the function describing the driver's acceleration strategy based on traffic states. The dynamics of the CAV are

$$\dot{s}_0(t) = v_{-1}(t) - v_0(t), \tag{3}$$
$$\dot{v}_0(t) = u(t-\tau), \tag{4}$$

with $v_{-1}(t)$ being the speed of the HHV, u being the control input, and $\tau > 0$ being the control input delay. In practice, multiple types of delay exist for CAV control: the onboard sensor measures environment state with detection delay; the vehicle-to-vehicle or vehicle-to-infrastructure wireless information exchange brings transmission delay; the engine and brake actuator have response delay [4], [23], just to name a few. These types of delay can be formulated as τ in the control input [22].

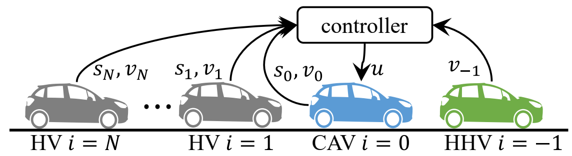


Fig. 1. We consider the control of a platoon of one CAV, one head HV, and N following HVs. The control input u is actuated to the CAV with delay $\tau > 0$. And the disturbance comes from the speed of HHV v_{-1}, which evolves independently from the control input u.

The steady-state vehicles drive at the same equilibrium speed v^*, and the equilibrium gap s_i^* of each vehicle i is decided by $F_i(s_i^*, 0, v^*) = 0$. Define state variations around the equilibrium spacing s_i^* and speed v^*,

$$\tilde{s}_i(t) = s_i(t) - s_i^\star, \tag{5}$$
$$\tilde{v}_i(t) = v_i(t) - v^\star, \tag{6}$$

the linearized dynamics for the HV are obtained from (1)-(2) as

$$\dot{\tilde{s}}_i(t) = \tilde{v}_{i-1}(t) - \tilde{v}_i(t), \tag{7}$$
$$\dot{\tilde{v}}_i(t) = a_{i1}\tilde{s}_i(t) - a_{i2}\tilde{v}_i(t) + a_{i3}\tilde{v}_{i-1}(t), \tag{8}$$

where

$$a_{i1} = \frac{\partial F_i}{\partial s_i}, \quad a_{i2} = \frac{\partial F_i}{\partial \dot{s}_i} - \frac{\partial F_i}{\partial v_i}, \quad a_{i3} = \frac{\partial F_i}{\partial \dot{s}_i}, \tag{9}$$

are evaluated at the steady states $(s_i^*, 0, v^*)$. And the linearized dynamics for the CAV are

$$\dot{\tilde{s}}_0(t) = \tilde{v}_{-1}(t) - \tilde{v}_0(t), \tag{10}$$
$$\dot{\tilde{v}}_0(t) = u(t-\tau). \tag{11}$$

The state of the linearized mixed autonomy traffic model is

$$x = \left[\tilde{s}_0,\ \tilde{v}_0,\ \tilde{s}_1,\ \tilde{v}_1, \cdots, \tilde{s}_N,\ \tilde{v}_N\right]^\top \in \mathbb{R}^n, \tag{12}$$

with $n = 2N+1$. The linearized system model with actuation delay is

$$\dot{x}(t) = Ax(t) + Bu(t-\tau) + Dr(t), \tag{13}$$

with the disturbance $r(t) = \tilde{v}_{-1}(t)$, and the coefficients being

$$A = \begin{bmatrix} P_0 & & & \\ Q_1 & P_1 & & \\ & \ddots & \ddots & \\ & & Q_N & P_N \end{bmatrix}, B = \begin{bmatrix} b_0 \\ 0_{2\times1} \\ \vdots \\ 0_{2\times1} \end{bmatrix}, D = \begin{bmatrix} d_0 \\ 0_{2\times1} \\ \vdots \\ 0_{2\times1} \end{bmatrix}, \tag{14}$$

where

$$P_0 = \begin{bmatrix} 0, & -1 \\ 0, & 0 \end{bmatrix}, P_i = \begin{bmatrix} 0, & -1 \\ a_{i1}, & -a_{i2} \end{bmatrix}, Q_i = \begin{bmatrix} 0, & 1 \\ 0, & a_{i3} \end{bmatrix}, b_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, d_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

B. Nominal feedback controller without input delay

A nominal controller for the platoon (13) without delay is given by the feedback law considering the states of both leading and following vehicles [21],

$$\begin{aligned} u_0(t) &= a_1\tilde{s}_0(t) - a_2\tilde{v}_0(t) + a_3\tilde{v}_{-1}(t) + \sum_{i=1}^{N}(\mu_i\tilde{s}_i(t) + k_i\tilde{v}_i(t)) \\ &= Kx(t) + a_3 r(t), \end{aligned} \tag{15}$$

with a feedback gain $K = [\alpha_1, -\alpha_2, \mu_1, k_1, \cdots, \mu_N, k_N]$.

The string stability of a platoon is introduced to describe the attenuation of perturbations from the leading vehicle to the following vehicle. The definition is given as follows.

Definition 1 ([21] Head-to-tail string stability). *A platoon is head-to-tail string stable if and only if*

$$|\Gamma(j\omega)|^2 \leq 1, \quad \omega \geq 0, \tag{16}$$

where the head-to-tail transfer function is

$$\Gamma(s) = \frac{\widetilde{V}_N(s)}{\widetilde{V}_{-1}(s)}, \tag{17}$$

with $\widetilde{V}_N(s)$ and $\widetilde{V}_{-1}(s)$ being the Laplace transform of speed perturbation of tail HV-N and head HV -1 respectively.

For the system (13) with the feedback controller in (15), by choosing the feedback gain K to satisfy (16), the platoon is string stable. But this nominal controller can violate safety (15), namely causing rear-end collisions. We design a safety-critical controller to guarantee safety in the next section.

III. Delay-Compensating Safety-Critical Traffic Control

In this section, we introduce the proposed RSTC controller to guarantee safety under input delay and disturbances. We present and prove the safety constraints in Theorem 2.

A. State predictor and delay compensation

Given the mixed autonomy traffic model (13), the state at $t+\tau$ is decided by the state at t, the historical input u from $t-\tau$ to t, and the disturbance r from t to $t+\tau$ as

$$\begin{aligned} x(t+\tau) =& \mathrm{e}^{A\tau}x(t) + \int_0^\tau \mathrm{e}^{A(\tau-\theta)}Bu(t+\theta-\tau)\mathrm{d}\theta \\ &+ \int_0^\tau \mathrm{e}^{A(\tau-\theta)}Dr(t+\theta)\mathrm{d}\theta. \end{aligned} \tag{18}$$

The future speed of HHV is typically unavailable, and thus we design a predictor to get the predicted state $x_p(t)$ as

$$\begin{aligned} x_p(t) =& \mathrm{e}^{A\tau}x(t) + \int_0^\tau \mathrm{e}^{A(\tau-\theta)}Bu(t+\theta-\tau)\mathrm{d}\theta \\ &+ \int_0^\tau \mathrm{e}^{A(\tau-\theta)}Dr(t)\mathrm{d}\theta. \end{aligned} \tag{19}$$

To compensate for the input delay, we use the predicted state $x_p(t)$ in (19) to get the nominal controller as

$$u_0(t) = Kx_p(t) + a_3 r(t). \tag{20}$$

Since we mainly focus on safe guarantee of the delayed control, the string stability of the controller (20) with inaccurate prediction and disturbance is beyond the scope of this paper and can be found in [9].

B. Safe spacing policy

To avoid rear-end collisions, we adopt the straightforward and common Constant Time Headway (CTH) spacing policy. For each vehicle i, its gap should satisfy

$$s_i \geq \psi_i v_i, \quad \forall i = 0, 1, \cdots, N, \tag{21}$$

with $\psi_i > 0$ being the time headway. The safe set for each vehicle is

$$\mathscr{C}_i = \{x \in \mathbb{R}^n : h_i(x) \geq 0\}, \tag{22}$$

with

$$h_i(x) = s_i - \psi_i v_i = \tilde{s}_i - \psi_i\tilde{v}_i + s^* - \psi_i v^*. \tag{23}$$

To meet the safety constraints (21), we adopt the control barrier function.

Definition 2 ([3]Control barrier function). *Consider an affine control system*

$$\dot{x} = f(x) + g(x)u, \tag{24}$$

with $x \in \mathscr{D} \subset \mathbb{R}^{n_1}$, $u \in \mathscr{U} \subset \mathbb{R}^{n_2}$, f and g locally Lipschitz, let $\mathscr{C}$ be the superlevel set of a continuously differentiable function $h : \mathscr{D} \to \mathbb{R}$, the function h is called a control barrier function for the system (24) on $\mathscr{C}$ if there exists an extended class $\mathscr{K}_\infty$ function α such that

$$\sup_{u\in\mathscr{U}} L_f h(x) + L_g h(x)u \geq -\alpha(h(x)), \quad \forall x \in \mathscr{D}, \tag{25}$$

with $L_f h = \nabla h(x)\cdot f(x)$ and $L_g h = \nabla h(x)\cdot g(x)$ being the Lie derivatives.

The safety constraint is guaranteed by CBF in Theorem 1.

Theorem 1. *[3] If h is a control barrier function and $\frac{\partial h}{\partial x} \neq 0$ for all $x \in \partial\mathscr{C}$, then any Lipschitz continuous controller u satisfies*

$$L_f h(x) + L_g h(x) \geq -\alpha(h(x)), \tag{26}$$

renders the set $\mathscr{C}$ forward invariant, i.e, for all $x(0) \in \mathscr{C}$, $x(t) \in \mathscr{C}$.

When there is no input delay in the mixed autonomy model (13), i.e., $\tau = 0$, CBF guarantees safety with $f(x) = Ax + Dr$ and $g(x) = B$. When there is an input delay, we derive safety constraints in the next section.

C. Safety by robust CBF under prediction error

We first make the following assumptions.

Assumption 1. *The system is safe before the control input $u(0)$ is actuated, i.e., $x(t) \in \mathscr{C}_i$ holds for all $t \in [0, \tau)$ and for all $i = 0, 1, \cdots, N$.*

Assumption 2. *For the disturbance $r(t)$, its derivative $\dot{r}(t)$ is bounded*

$$\underline{a} \leq \dot{r}(t) \leq \bar{a}, \quad \forall t \geq 0. \tag{27}$$

Since $r(t)$ is the speed perturbation of the leading vehicle, Assumption 2 indicates the accelerations and decelerations of the leading vehicle are physically bounded by $\bar{a}$ and $\underline{a}$, which always hold in practice.

Theorem 2 (Safety for mixed autonomy)**.** *Under Assumption 1 and Assumption 2, for the mixed autonomy system* (13)*, if a Lipschitz continuous controller $u(t)$ satisfies*

$$L_f h_0(x_p(t)) + L_g h_0(x_p(t))u(t) \geq -\alpha_0(h_0(x_p(t))) + M_0(t), \tag{28}$$

where $f(x(t)) = Ax(t)$, $g(x(t)) = B$, $x_p(t)$ is the predicted state by (19)*, α_0 is an extended class $\mathscr{K}_\infty$ function, the function $M_0(t) : \mathbb{R}^+ \to \mathbb{R}$ reflects the effect of delay τ and disturbances on safety and is defined as*

$$\begin{aligned} M_0(t) =& \alpha_0(h_0(x_p(t))) - \alpha_0(h_0(x_p(t)) + \underline{a}\tau^2/2) \\ & - (r(t) + \underline{a}\tau), \end{aligned} \tag{29}$$

then the safe set for CAV $\mathscr{C}_0$ (22) *is forward invariant. If u also satisfies*

$$L_f h_i^{\mathrm{r}}(x_p(t)) + L_g h_i^{\mathrm{r}}(x_p(t))u(t) \geq -\alpha_i(h_i^{\mathrm{r}}(x_p(t))) + M_i(t), \tag{30}$$

where α_i is an extended class $\mathscr{K}_\infty$ function, the function $h_i^{\mathrm{r}} : \mathbb{R}^n \to \mathbb{R}$ is defined as:

$$h_i^{\mathrm{r}}(x) = h_i(x) - \eta_i h_0(x) - \eta_i \underline{a}\tau^2/2, \tag{31}$$

with $\eta_i > 0$, and the function $M_i(t) : \mathbb{R}^+ \to \mathbb{R}$ is defined as

$$\begin{aligned} M_i(t) =& \alpha_i(h_i^{\mathrm{r}}(x_p(t))) - \alpha_i(h_i^{\mathrm{r}}(x_p(t)) - \underline{a}\eta_i\tau^2/2) \\ & + \eta_i(r(t) + \bar{a}\tau), \end{aligned} \tag{32}$$

then the safe set for HV-i $\mathscr{C}_i$ (22) *is forward invariant.*

Proof. We first prove the safety constraint for the CAV (28). For the predictor (18), the prediction error is

$$\begin{aligned} x_e &= x(t+\tau) - x_p(t) \\ &= \int_0^\tau e^{A(\tau-\theta)} D(r(t+\theta) - r(t)) \mathrm{d}\theta. \end{aligned} \tag{33}$$

From the model coefficients in (14), we have

$$e^{A(\tau-\theta)} D = D = [1, 0, \cdots, 0]^\top, \tag{34}$$

which indicates that there is only a prediction error for the CAV's gap. Denote $s_{0,p}(t)$ and $v_{0,p}(t)$ as the predicted gap and speed for CAV, we have from Assumption 2:

$$s_0(t+\tau) - s_{0,p}(t) = \int_0^\tau (r(t+\theta) - r(t)) \, \mathrm{d}\theta \geq \frac{\underline{a}\tau^2}{2}, \tag{35}$$

$$v_0(t+\tau) - v_{0,p}(t) = 0. \tag{36}$$

By the prediction error (35)-(36), we see that to meet the safe gap condition in (21), it is sufficient to satisfy

$$s_{0,p}(t) + \underline{a}\tau^2/2 \geq \psi_0 v_{0,p}(t), \tag{37}$$

If we define a robust safety function $h_{0R}(x) : \mathbb{R}^n \to \mathbb{R}$ as

$$h_{0R}(x) = h_0(x) + \underline{a}\tau^2/2, \tag{38}$$

then we have $h_{0R}(x_p(t)) \geq 0 \implies h_0(x(t+\tau)) \geq 0$.

The dynamics of the predictor is

$$\dot{x}_p(t) = Ax_p(t) + Bu(t) + Dr(t) + D\tau\dot{r}(t). \tag{39}$$

By the definition of $h_{0R}(x)$ and $h_0(x)$, we have

$$\frac{\partial h_{0R}(x_p)}{\partial x_p} = \frac{\partial h_0(x_p)}{\partial x_p} = \begin{bmatrix} H_0 & 0_{1\times2} \cdots 0_{1\times2} \end{bmatrix}^\top, \tag{40}$$

with $H_0 = [1, -\psi_0]$. So the time derivative of $h_{0R}(x_p(t))$ is:

$$\begin{aligned} \dot{h}_{0R}(x_p(t)) =& L_f h_0(x_p(t)) + L_g h_0(x_p(t))u(t) \\ & + \frac{\partial h_0(x_p(t))}{\partial x_p} D(r(t) + \tau\dot{r}(t)). \end{aligned} \tag{41}$$

To guarantee $h_{0R}(x_p(t)) \geq 0$, the controller should satisfy

$$\dot{h}_{0R}(x_p(t)) \geq -\alpha_0(h_{0R}(x_p(t))). \tag{42}$$

However, since $\dot{r}(t)$, the acceleration of the head vehicle is unknown, $\dot{h}_{0R}(x_p(t))$ is also unknown. From the model coefficient D in (14), we have

$$\frac{\partial h_0(x_p(t))}{\partial x_p} D = 1 > 0. \tag{43}$$

So we have

$$\begin{aligned} \dot{h}_{0R}(x_p(t)) \geq & L_f h_0(x_p(t)) + L_g h_0(x_p(t))u(t) \\ & + r(t) + \tau\underline{a}, \end{aligned} \tag{44}$$

where $\underline{a}$ is a bound on $\dot{r}$ in Assumption 2. To meet (42), it is sufficient to have (28).

Now we prove the safety constraints for HVs (30). By the prediction error derived in (33) and (34), we have that there is no prediction error for HV-i's gap and speed, and $h_i(x_p(t)) \geq 0 \implies h_i(x(t+\tau)) \geq 0$. For h_i, it has a relative degree $i+1$ with respect to the control input. While high-order CBF can be designed [24], it usually requires a more complex formulation that is sensitive to parameters [3]. To avoid this, we design a reduced-order CBF candidate with a relative degree one for the HV as h_i^{r} in (31). It is straightforward that if $h_{0R}(x_p) \geq 0$ and $h_i^{\mathrm{r}}(x_p) \geq 0$, then $h_i(x_p) \geq 0$, which implies that the original safe criterion $h_i(x(t+\tau))$ is met.

The time derivative of $h_i^{\mathrm{r}}(x_p(t))$ is

$$\begin{aligned} \dot{h}_i^{\mathrm{r}}(x_p(t)) =& L_f h_i^{\mathrm{r}}(x_p(t)) + L_g h_i^{\mathrm{r}}(x_p(t))u(t) \\ & + \frac{\partial h_i^{\mathrm{r}}(x_p)}{\partial x_p} D(r(t) + \tau\dot{r}(t)). \end{aligned} \tag{45}$$

where

$$\frac{\partial h_i^{\mathrm{r}}(x_p)}{\partial x_p} = \begin{bmatrix} -\eta_i H_0 & 0_{1\times2} & \cdots & H_i & \cdots & 0_{1\times2} \end{bmatrix}^\top, \tag{46}$$

with $H_i = [1, -\psi_i]$. To guarantee $h_i^{\mathrm{r}}(x_p(t)) \geq 0$, the control input u should satisfy

$$\dot{h}_i^{\mathrm{r}}(x_p(t)) \geq -\alpha_i(h_i^{\mathrm{r}}(x_p(t))). \tag{47}$$

We note that

$$\frac{\partial h_i^{\mathrm{r}}(x_p)}{\partial x_p} D = -\eta_i < 0, \tag{48}$$

so a sufficient condition for (47) is (30). ■

TABLE I

ACCELERATION OF VEHICLES

	Scenario 1	Scenario 2
HHV	a_H if $t \in [5, 5+t_H]$; $-a_H$ if $t \in (5+t_H, 5+2t_H]$; 0 otherwise	0
CAV	(20) for nominal controller (49) RSTC	(20) for nominal controller (49) RSTC
HV1	(2)	(2)
HV2	(2)	a_F if $t \in [5, 5+t_F]$ (2) otherwise RSTC

We note that there could be a conflict between safety constraints for CAV (28) and for HVs (30). To make the problem feasible, we prioritize safety for the CAV by setting (28) as a hard constraint and (30) as soft constraints. This can be justified by the following three reasons: the controller carries primary responsibility for CAV safety; a collision on CAV also affects HVs but not vice versa; human drivers will take active actions to avoid collisions in risky scenarios. We formulate the QP (49) to solve a safety-critical control input:

$$u = \underset{u\in\mathbb{R},\sigma_i\geq 0}{\operatorname{argmin}} \; |u-u_0|^2 + \sum_{i=1}^{N} p_i \sigma_i^2 \tag{49}$$

s.t.

$$L_f h_0(x_p) + L_g h_0(x_p) u + \alpha_0(h_0(x_p)) + M_0 \geq 0,$$
$$L_f h_1^{\mathrm{r}}(x_p) + L_g h_1^{\mathrm{r}}(x_p) u + \alpha_1(h_1^{\mathrm{r}}(x_p)) + M_1 + \sigma_1 \geq 0,$$
$$\vdots$$
$$L_f h_N^{\mathrm{r}}(x_p) + L_g h_N^{\mathrm{r}}(x_p) u + \alpha_N(h_N^{\mathrm{r}}(x_p)) + M_N + \sigma_N \geq 0,$$

with $x_p = x_p(t)$ being the predicted state by (19), σ_i being slack variables, and p_i being penalty coefficients.

In (49) although we include slack variable σ_i for HVs, the safety constraint for the CAV is satisfied. In the related work [8], [15], the safety constraint for the CAV is also relaxed, which means there can be safety violations for the CAV.

IV. SIMULATION

In this section, we run numerical simulations to demonstrate the safety guarantee of the RSTC controller.

A. Simulation setting

We consider a platoon of $N = 4$ HVs. For the car-following model $F(s, \dot{s}, v)$ in (2), we use the Optimal Velocity Model (OVM),

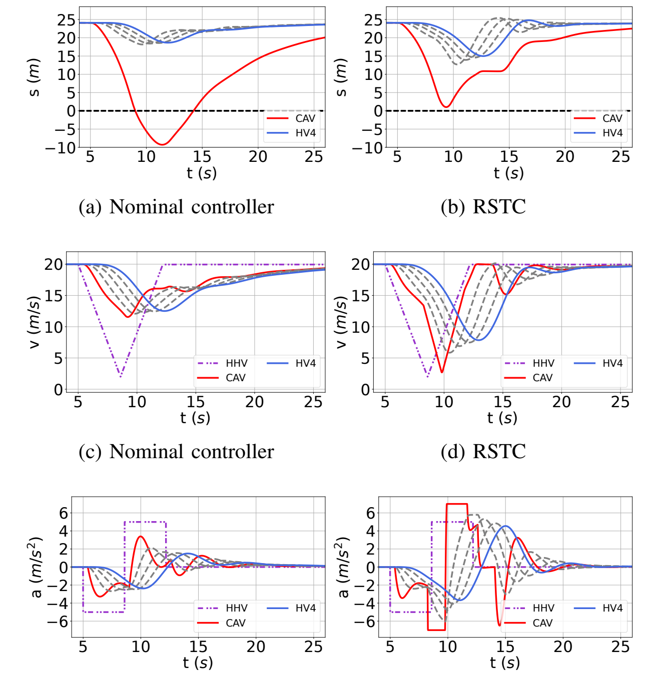
$$F_i(s_i, \dot{s}_i, v_i) = a\left(V(s_i) - v_i\right) + b\dot{s}_i, \tag{50}$$

where $V(s)$ is the desired speed-gap relationship as

$$V(s) = \begin{cases} 0, & s \leq s_{\mathrm{st}}, \\ \frac{v_{\max}}{2}\left(1 - \cos\left(\pi \frac{s - s_{\mathrm{st}}}{s_{\mathrm{go}} - s_{\mathrm{st}}}\right)\right), & s_{\mathrm{st}} < s < s_{\mathrm{go}}, \\ v_{\max}, & s \geq s_{\mathrm{go}}, \end{cases} \tag{51}$$

We take the parameters as $a = 0.6$, $b = 0.9$, $s_{\mathrm{st}} = 5$ m, $s_{\mathrm{st}} = 40$ m, and $v_{\max} = 35$ m/s. We set the equilibrium speed as $v^* = 20$ m/s, and the the equilibrium gap is decided by $V(s^*) = v^*$ as $s^* = 24$ m.

We set the delay as $\tau = 0.4$ s. For the nominal controller (20), we set the feedback gain as $\mu_i = -2$ and $k_i = 0.2$.



(a) Nominal controller (b) RSTC

(c) Nominal controller (d) RSTC

(e) Nominal controller (f) RSTC

Fig. 2. Profile of gap, speed, and acceleration of the nominal controller and RSTC controller in Scenario 1.

For the safe spacing policy CTH (21), we take the time headway as $\psi_0 = 0.5$ s for the CAV and $\psi_i = 1$ s for following HVs.

We consider two practical safety-critical traffic scenarios that can cause rear-end collisions in practice:

- Scenario 1: The HHV suddenly decelerates. This happens when there is cut-in behavior or obstacles on the road.
- Scenario 2: The HV suddenly accelerates. This can cause due to fatigue driving or wrong operations of the human driver.

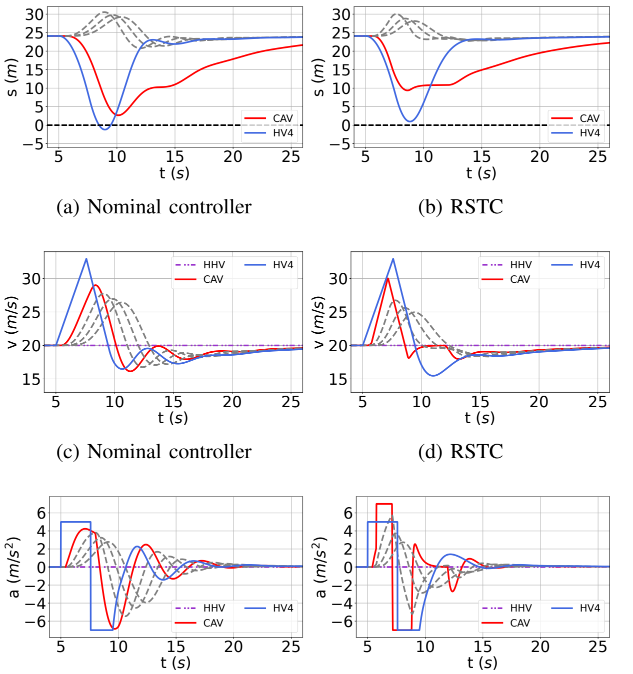
The acceleration of the four vehicles is listed in Table I.

B. Safety improvement

For Scenario 1, we set the deceleration of HHV as $a_H = 5$ m/s^2 with a duration of $t_H = 3.6$ s. In Fig. 2, we plot the gap s, speed v, and acceleration a of the platoon got by the nominal controller (20) and the proposed RSTC controller (49). We see that when the HHV decelerates, the nominal controller gives smaller deceleration, which makes the platoon string stable but causes a collision, i.e., $s_0 < 0$ as in Fig. 2(a). The proposed RSTC controller gives a larger deceleration to avoid the collision. In Fig. 3, we give the gap, speed, and acceleration profile in Scenario 2 with $a_F = 5$ m/s^2 and $t_F = 2.6$ s. Similarly, the RSTC controller improves safety and avoids the rear-end collision.

V. CONCLUSION

In this paper, we propose a robust safety-critical traffic controller for mixed autonomy with input delay and external



(a) Nominal controller (b) RSTC

(c) Nominal controller (d) RSTC

(e) Nominal controller (f) RSTC

Fig. 3. Profile of gap, speed, and acceleration of the nominal controller and RSTC controller in Scenario 2.

disturbance. A state predictor is adopted to predict the system state with bounded error. The CBF is utilized to construct safety constraints for CAV and HVs. By simulation, we show that the proposed RSTC controller avoids rear-end collision and allows for a larger safe range of speed perturbation in the platoon. Future work of interest includes considering state delay and the safe controller with multiple CAVs.

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