Systems & Control: Foundations & Applications

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Preface

This book explores the development of PDE (partial differential equation) backstepping controllers for the suppression of stop-and-go instabilities and oscillations in congested traffic flows. As such, the book should be considered at least as much as an addition to the literature on boundary control of coupled systems of first-order hyperbolic PDEs as an addition to the traffic control literature.

The motivation for the work comes from traffic dynamics in the congested regime. It is only in the congested regime, when the vehicles are relatively close by, that stop-and-go oscillations develop. Stop-and-go is simply the consequence of sensory and cognitive limitations of the humans participating in traffic. Every driver has a reaction delay that is on the order of a second. Moreover, an average human driver is not even able to predict the dynamic response of their own vehicle to the accelerator and brake pedal inputs that they apply, let alone predicting the motion of the vehicle in front of them, and even much less the motion of the whole chain of vehicles further ahead. In conclusion, as long as operating their own vehicles is an option that humans are free to exercise, stop-and-go oscillations will be a part of traffic flows.

Stop-and-go does not only carry a high cost in comfort and frustration. Societal costs in traffic safety, the additional fuel consumption, and the total driving time are enormous. These are the reasons that make even partial suppression of stop-and-go oscillations an important pursuit.

For us, personally, the importance of this pursuit is not less worthy due to the complex control design task of developing feedback laws for the extremely high-dimensional traffic flow systems, whose neatest modeling is in the form of PDEs, and which, in spite of their high dimensionality, have to be controlled using one or very few inputs, such as ramp metering or variable speed limits. That one input, or those few inputs, is located at boundaries of freeway segments, and the entire traffic between the locations of such inputs, which involves hundreds, and sometimes thousands of vehicles, needs to be controlled from the boundary. It is this type of a control design challenge that attracts PDE control designers to traffic flows.

While variable speed limit signs are not uncommon in certain metropolitan areas, ramp metering, applied with traffic lights that let on only one or two cars onto the freeway, at certain time intervals (on the order of seconds, or tens of seconds), are very common. By not keeping the length of those time intervals constant, but by varying them instead, it is possible to vary the flow of traffic at the location of the on-ramp.

An increased flow at the on-ramp influences the drivers approaching the on-ramp on the freeway mainline to respond to the increased flow from the on-ramp by slowing down. In particularly concrete terms, the following process takes place: vehicles entering from the on-ramp are trying to merge into the freeway traffic flow; drivers on the freeway observe the vehicles entering the freeway on their right; drivers on the mainline freeway respond to the fact that vehicles are merging from the on-ramp by adjusting their speeds, in order to avoid a collision with the merging vehicles.

So, the modulation of the duration of the red and green lights on the on-ramp influences the speed, and the density, of the vehicles immediately upstream of the on-ramp. How can this control capability of the on-ramp traffic lights be exploited to impact the traffic flow a great distance away from the on-ramp?

Four possibilities exist in this regard. Ramp metering can be employed to control traffic both downstream and upstream. And the traffic on either freeway segment (downstream or upstream of the on-ramp) can be in either the free or the congested regime. So, there exists a set of 2×2, namely four combinations of traffic flow possibilities.

Among those four combinations, one is not controllable (traffic upstream of the on-ramp in the free regime); two are trivial, requiring either no control (traffic downstream of the on-ramp in free regime) or a simple collocated proportional feedback (traffic downstream of the ramp in the congested regime); and only one of these four cases is challenging and truly interesting: control of traffic flow upstream of the on-ramp in the congested regime.

How can one possibly control the dynamics of traffic *upstream* of an on-ramp by varying the duration of the on-ramp's traffic lights? In the congested regime, this is possible. In the congested regime, the vehicles are relatively close by, and the drivers respond to the changes in the distances relative to the cars immediately in front of them. These responses are "chain reactions," which propagate upstream. In intuitive terms, in congested traffic, the reactions of drivers to the variations in distance progress in the upstream direction faster than the cars move in the downstream direction. It is due to this effect, of a "human behavioral wave" (of braking or accelerating), that modulations of a traffic light at an on-ramp can affect the traffic dynamics a long distance upstream of the ramp.

The colloquial depiction of the possibility of controlling congested traffic upstream of an on-ramp also has its formal, rigorous representation in PDE models of traffic. The suitable model for stop-and-go oscillations is the Aw-Rascle-Zhang (ARZ) model. This model incorporates two coupled nonlinear first-order hyperbolic PDEs, one for the density and one for velocity. In such a model with two hyperbolic PDEs, two waves arise, which propagate in certain directions and at certain speeds. It happens so that in the free traffic, the two waves both propagate in the downstream direction, whereas in the congested traffic, one wave propagates in the downstream direction whereas the other wave propagates in the upstream direction.

It is this wave in the system of two hyperbolic PDEs, which propagates in the upstream direction in the congested regime, that is related to the drivers' responses to the distance fluctuations and that runs "backward," being passed to the driver behind the reacting driver.

In summary, in the congested regime, the ARZ PDE model of traffic is controllable by ramp metering in the upstream direction. It is this property that we build on, advancing from the mere controllability to a feedback design methodology for stabilizing the stop-and-go oscillations.

Our feedback design is based on the method of *PDE backstepping*. While PDE control can be approached also with linear quadratic (LQR, LQG) methods, as well as methods based on pole placement for reduced models of PDEs, backstepping has the advantages of neither requiring model reduction of the PDE plant nor the approximation of solutions of nonlinear infinite-dimensional equations such as operator Riccati equations.

Backstepping, introduced by the second author, with early contributions by his student Andrey Smyshlyaev and postdocs Weijiu Liu and Andras Balogh, initially for parabolic PDEs, employs a spatial Volterra transformation to convert the plant PDE, which may be unstable, into a "target system." The target system is chosen by the designer, as a PDE of a relatively simple and familiar form, within the same class as the plant PDE. By its very selection, the target system's stability is relatively easy to establish.

The Volterra kernel of the backstepping transformation is a linear PDE of the Goursat form which can be transformed from a PDE into an integral equation. In its integral equation form, this transformation kernel can be computed fairly easily, using successive approximations. The existence and uniqueness of solutions to this kernel PDE can be established much more easily than for operator Riccati equations or other standard PDEs that are not in the Goursat form.

It is for these reasons that PDE backstepping has seen a wide adoption, both among theorists and in PDE control applications. The applications are numerous, and we leave out the common strings and beams but emphasize turbulent flows; water canals; phase change in materials, including thermal dynamics with liquid-solid transitions, as in 3D printing; and industrial applications in state estimation for lithium-ion batteries and in oil drilling.

The interest in developing backstepping controllers for traffic began in 2007, with the one-semester stay of the second author at the University of California, Berkeley, where he taught a course on PDE backstepping control. At that time, backstepping was not yet developed for hyperbolic PDEs. The breakthrough in this development took place in the 2013 paper on PDE backstepping for 2×2 hyperbolic systems by Coron, Vazquez, Krstic, and Bastin.

Our work on applying PDE backstepping for hyperbolic PDEs to traffic flows began in 2016 with the development of an adaptive control design for the ARZ model of traffic with unknown parameters. Backstepping makes parameter-adaptivity viable in PDE control due to the nearly explicit form of the gain functions.

This and other results in this book have been developed in the course of the first author's doctoral and postdoctoral research at UC San Diego with the second author, from 2016 until 2021.

What Does the Book Cover?

If there is anything we hope that the reader will learn from this book, it is how to design controllers that suppress oscillations and even instabilities in traffic flow. The introduction to those tools is contained in Chap. 3. An extension from single-lane to multi-lane traffic is in Chap. 8 and from single-class to multi-class traffic in Chap. 9.

Large uncertainties in traffic models can arise due to weather and visibility conditions, as well as due to many other reasons which make the classes of drivers and vehicles vary with the time of day and other environmental and societal factors. Stabilization of traffic using learning-based methods is covered in Chap. 5 (using adaptive control) and Chap. 7 (using reinforcement learning).

Implementations of the stop-and-go suppressing controllers in event-triggered fashions are presented in Chap. 6.

Traffic-routing apps can make the driving behaviors dependent not only on the flow and density states immediately in front of the driver but on the flow and density distributions along the entire freeway, which is referred to as "non-local feedback" and can exacerbate the stop-and-go motions. Chapter 12 extends the PDE backstepping designs from routing-free traffic flows to flows that incorporate routing. This chapter provides control laws which mitigate the non-local effects of the routing apps.

Even when the infrastructure lacks the means of actuating the traffic (ramp metering, variable speed limits, etc.), it is of interest to perform a real-time estimation of the traffic flow along the freeway using measurements at only a few locations. Observers, i.e., state estimator designs for traffic flows modeled by ARZ PDEs, are presented in Chaps. 3, 4, and 11.

While most of the book deals with potentially unstable, or at least oscillatory, traffic, and therefore with traffic modeled by second-order PDEs (ARZ), in two chapters we deal with problems for first-order PDE models (LWR). Chapter 13 shows how a shock which delineates the boundary between congested traffic (in the front) and free traffic (in the back, i.e., "incoming" traffic) can be regulated to a desired location.

In Chap. 14, we show how the LWR traffic flow can be maximized through a bottleneck that may arise in an unplanned fashion, such as from a traffic accident. In such a case, maximizing the flow rate through the bottleneck has to be pursued without a reliance on a model of the flow rate versus density relationship at the bottleneck. We maximize the flow using the extremum seeking approach and a predictor-based compensation of the LWR dynamics.

To make the book self-contained, we incorporate Chap. 2, which contains an introduction to the PDE backstepping method for systems of coupled first-order hyperbolic PDEs.

Who Is the Book For?

This book deals with control theory, dynamical systems, partial differential equations, and traffic flows. In integrating these subjects, it may offer material of interest to readers who conduct research in these areas and have training in areas as diverse as electrical engineering, mechanical engineering, civil engineering and transportation, applied mathematics, applied physics, and even machine learning and computer science.

In dealing with suppression of traffic instability, the primary audience of this research monograph are control theorists working on control of systems modeled by PDEs and traffic engineers and applied scientists working unsteady traffic flows.

For PDE control theorists, especially those focusing on feedback design and stability analysis, this book provides an entry point into one of the most exciting application areas for PDE control, especially for application of PDE control of hyperbolic PDEs. While coupled hyperbolic PDEs arise in many relevant sub-areas of flow control, including control of compressible fluids, such as those that are encountered in oil drilling, control of traffic flows offers as much technical challenge as any applications in one-dimensional fluid dynamics, while carrying the potential for orders of magnitude more in societal impact. In fact, stabilization of unsteady congested traffic is arguably the application of PDE control that is the most relatable for a lay audience. We do not know any PDE control application for which a non-expert has a comparable level of intuition and unequivocal belief in its importance. For this reason, we are hopeful that this book will serve to advance the interest in PDE control, and especially boundary control of PDEs, beyond the specific content of this book and our own work.

For a PDE control theorist interested in the range of capabilities of the PDE backstepping method, we particularly recommend Chap. 9 in which, in the context of a two-class traffic (such as a mix of large/inert and small/agile vehicles, or a mix of defensive and aggressive drivers), a coupled hyperbolic heterodirectional structure of the form $(3 + 1) \times (3 + 1)$ arises, in which a boundary control input (by ramp metering) is available only in one PDE channel, which convects in the upstream direction, whereas the three unactuated PDE channels convect in the downstream direction. This is a good example of the capability of PDE backstepping to stabilize a system of four PDEs using a single boundary input.

For traffic engineers and scientists, this book provides tools that have been previously unavailable for suppressing stop-and-go oscillations in congested traffic using actuation that is very sparsely located along the freeway, such as ramp metering or variable speed limits. During the next one or several decades, until automation of vehicles and their connectivity (such as in CAVs—connected and automated vehicles) achieves sufficient levels of penetration (the pace of which appears a lot slower than the predictions of about a decade ago), and until such vehicle-level automation makes the suppression of traffic flow instabilities easier to achieve, the advanced and fairly complex methods introduced by this book will be an important option for the designer of traffic flow management systems.

PDE backstepping, the method upon which the designs presented in this book rely, is not simple to learn even for advanced control theorists. So, we do not expect effortless adoption of these methods by traffic engineers, whose core training in control may be around methods like LQR or basic nonlinear control for finite-dimensional systems. Cognizant of this challenge, we have made our exposition as accessible as possible, as self-contained as possible, and stripped of mystifying conventions that are common in the exposition of material within mathematical fields like analysis of PDEs and PDE control. This deliberate commitment to accessibility deprives the more mathematical specialist in PDE control of nearly nothing that they will not infer themselves from the context.

Parts of the book will be of interest to control engineers who do not intend to specialize in PDE control but specialize in other areas. For example, a specialist in extremum seeking will be inspired by the role this model-free optimization methodology has to play in traffic control by reading Chap. 14. Likewise, a specialist in delay systems will find it revealing that predictor-based feedback designs, for compensation of input delays, are the key ingredient for regulating the position of a moving shock in traffic density on a congested freeway by reading Chap. 13. A specialist in adaptive control will see how far the boundaries of this classical field can be taken by reading about adaptive control design for the ARZ PDE model of traffic in Chap. 5. A specialist in sampled-data control, used to studies in emulating continuous-time control designs for linear and nonlinear PDEs, will see how those techniques extend to PDE control, with the aid of ISS and small-gain theorems for PDEs, by reading Chap. 6.

Finally, for a specialist outside of the classical field of control theory—a reader interested in reinforcement learning and, more generally, machine learning and AI methods—the book offers, in Chap. 7, a thought-provoking comparison between model-based PDE control and learning-based acquisition of a similar capability through simulation-based training.

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