Safe Stabilizing Control of Traffic Systems With Simultaneous State and Actuator Delays

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***Abstract*—The integration of connectivity and autonomy in vehicles has shown great promise in smoothing traffic by a leading connected automated vehicle (CAV), which can be achieved by designing cruising control strategies to regulate the velocity and spacing of a string of follower vehicles. Safety verification of these stabilizing traffic controllers in real-world traffic systems, where delays stem from human reaction times and sensor/actuator latency, remains an open research question. In this letter, we solve the safe stabilization of a class of traffic systems with simultaneous state and actuator delays, using the control barrier function (CBF). A predictor with bounded prediction error is designed to compensate for the actuator delay and state delays. Delay-compensating CBF constraints are designed to guarantee formal safety under simultaneous state and actuator delays. We synthesize a safety-critical controller by solving a Quadratic Programming problem that minimizes deviation from a nominal stabilizing traffic controller. Numerical simulations discuss the safety impact of simultaneous reaction delay of human drivers and the actuator delay of CAV in two safety-critical scenarios and validate the proposed CBF safety constraints.**

***Index Terms*—Delay systems, traffic control, autonomous vehicles, constrained control.**

I. Introduction

USING connected and automated vehicles (CAVs) to stabilize platoon traffic and alleviate congestion has been widely studied by both theory [15] and field experiments [10]. Application of CAVs cover various traffic scenarios, such as intersections, on-ramp merging, platooning operations [2], [7], [11], [16]. One main challenge to promote CAV and increase public acceptance of CAV comes from safety concerns, especially the risk of rear-end collisions [9]. Traffic safety has been considered by some representative techniques including model predictive control [17], reachability analysis [6], and control barrier functions (CBFs) [19]. Among these methods, CBF imparts formal safety by synthesizing a safety-critical controller from modifying user-selected nominal controllers, and thus has a higher degree of freedom in design. Specifically, CBF is adopted to design safety-critical traffic control in the authors' work [19], which imparts a formal safety guarantee for CAV.

Previous work on vehicle-based traffic control has focused on delay-free traffic, while this letter considers time-delayed traffic. In practical traffic systems, there are intrinsic delays from multiple sources. For example, human drivers take action after a certain reaction time. The vehicle control system also contains delays, such as in the throttle actuator, engine response, or brake actuator [13]. For on-board sensors, there are delays caused by discrete sampling in the sensing/filtering process in measuring speed and gap. Those delays are lumped as state delay and actuator delay in traffic system [4].

Delays jeopardize the stability performance of controllers designed for delay-free systems. Analysis in [3] shows delays cause a smaller stability region of feedback controllers. While delay-robust stabilizing controllers have been designed [3], there still lacks a formal safety guarantee for time-delayed traffic systems. Safety-critical controllers designed for delay-free systems may cause safety violations under delays [8]. So we mainly focus on safe stabilization for delayed traffic systems in this letter.

Safety-critical controllers have been designed when there is only actuator delay or state delay. When there is only actuator delay, predictor-based CBF is developed, which constructs CBF constraints based on a predictor that predicts future state [8], [18]. Specifically, in the author's previous work [18], safety-critical controllers for CAV have been designed considering actuator delay. The CBF is extended to attain formal safety guarantee for systems with state delay in [5]. These methods cannot directly be applied to delayed traffic systems for two reasons. First, either state delay or actuator delay is considered, while both state and actuator delays present in traffic systems. And it remains an open question for safety guarantee design with both delays. Second, in traffic systems, the head vehicle's speed is an external disturbance that evolves independently from the control input, as in Fig. 1. This disturbance causes inaccurate prediction. For example, the gap between the CAV and its leading vehicle depends on the exact future value of the speed of these two vehicles. With unknown future speed of the leading vehicle over the delay interval,

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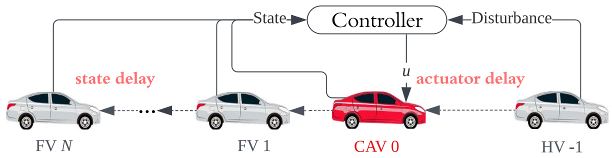

Fig. 1. Safe stabilizing of a vehicle chain under state delay and actuator delays is considered. The leader CAV follows a head vehicle (HV) with velocity disturbance and leads N follower vehicles (FVs) that present state delays.

the prediction of the gap thus becomes inaccurate. With an inaccurate prediction, the predictor-based CBF may still cause unsafe collisions with this inaccurate prediction [8]. This letter will focus on designing robust safety constraints and safe controller for traffic systems with state delay, actuator delay and disturbance.

To summarize, safety-critical control for delayed-traffic systems has yet to be addressed. In this letter, we consider safe stabilization of traffic systems with state delay, actuator delay, and disturbance. We first design a predictor for the compensation of the actuator delay, and prove that the predictor only has state delay in dynamics. We then derive an upper bound on the prediction error caused by disturbance and construct sufficient safety conditions on the predicted state. By utilizing CBF for systems with state delay, the predicted system remains safe, which means the origin delayed traffic system also remains safe.

The contribution of this result is twofold. On one hand, this letter presents the first theoretical result on safety-critical control for the linearized traffic systems with both actuator and state delays. The challenge of designing CBF lies in the coupling between the two types of delays. We propose a novel CBF safety design by embedding the distributed state delays into the predictor state for input delay compensation and then obtain a combined safety constraint accounting for both delays. Previous safety-critical control is considered for systems with input delay only [18] or state delay only [5]. On the other hand, we address the practical safety impact of simultaneous reaction delay of human drivers and the actuator delay of CAV in two safety-critical scenarios. The numerical results demonstrate that the proposed CBF design can still guarantee safety while the actuator-only result [18] may fail when considerate state delays are considered.

The remainder of this letter is organized as follows. We formulate the delayed traffic system in Section II. We design and prove robust CBF safety constraints in Section III and run simulations to validate the designed constraints in Section IV.

II. Time-Delayed Traffic Systems

We consider the longitudinal control for a string of vehicles as in Fig. 1, in which the CAV indexed as 0 follows a head vehicle (HV) indexed as -1 and leads N following vehicles (FVs) indexed from 1 to n.

For FV-i, we denote its speed as $v_i(t) \in \mathbb{R}$, and its spacing to its predecessor vehicle $i-1$ as $s_i(t) \in \mathbb{R}$. FV-i's dynamics are

$$\dot{s}_i(t) = v_{i-1}(t) - v_i(t), \tag{1}$$
$$\dot{v}_i(t) = F_i(s_i(t-\tau_{\mathrm{F}i}), v_i(t), v_{i-1}(t-\tau_{\mathrm{F}i})), \tag{2}$$

where $v_i(t) \in \mathbb{R}$ is speed of its leader vehicle. The FVs can be either human-driven vehicles or automated vehicles such as those equipped with adaptive cruise control. In the first case, the function F_i is the human driver's car-following behavior strategy. And the delay $\tau_{\mathrm{F}i}$ represents human reaction time. For the second case, the function F_i represents the cruising control of the vehicle, and the delay $\tau_{\mathrm{F}i}$ comes from sensors [4].

For the CAV, a controller control $u(t) \in \mathbb{R}$ is designed to control its longitudinal motion

$$\dot{s}_0(t) = v_{-1}(t) - v_0(t), \tag{3}$$
$$\dot{v}_0(t) = u(t-\tau_{\mathrm{U}}). \tag{4}$$

In traffic, there exist multiple types of delay, such as the detection delay from onboard sensor measurement, the communication from the information exchange between CAV and other vehicles via wireless communication, and the response delay from the powertrain. We use the actuator delay $\tau_{\mathrm{U}} > 0$ to incorporate those possible delays.

In this letter, we design safety constraints based on linearized systems. We will demonstrate the designed safety constraints also work for the non-linear system for numerical implementation (1)-(4). The safety design for non-linear system is left as future work. We linearized the system around the equilibrium states as follows. At the equilibrium states, the vehicle chain drives at a constant speed v^*. For FV-i, its gap with its leader remains constant as s_i^* given from $F_i(s_i^*, v^*, v^*) = 0$. For the CAV, its equilibrium gap s_0^* is set by the controller. We take perturbations as

$$\tilde{s}_i(t) = s_i(t) - s_i^\star, \tag{5}$$
$$\tilde{v}_i(t) = v_i(t) \ - v^\star. \tag{6}$$

The FVs' dynamics are linearized as

$$\dot{\tilde{s}}_i(t) = \tilde{v}_{i-1}(t) - \tilde{v}_i(t), \tag{7}$$
$$\dot{\tilde{v}}_i(t) = a_{i1}\tilde{s}_i(t-\tau_{\mathrm{F}i}) + a_{i2}\tilde{v}_i(t) + a_{i3}\tilde{v}_{i-1}(t-\tau_{\mathrm{F}i}), \tag{8}$$

where $a_{i1} = \frac{\partial F_i}{\partial s_i}, a_{i2} = \frac{\partial F_i}{\partial v_i}, a_{i3} = \frac{\partial F_i}{\partial v_{i-1}}$ are evaluated at the equilibrium state. We linearize the CAV's dynamics:

$$\dot{\tilde{s}}_0(t) = r(t) - \tilde{v}_0(t), \tag{9}$$
$$\dot{\tilde{v}}_0(t) = u(t-\tau_{\mathrm{U}}), \tag{10}$$

where $r(t) = \tilde{v}_{-1}(t)$ is taken as a disturbance from the speed perturbation of HV. We write the linearized perturbations compactly as

$$x = \left[\tilde{s}_0, \ \tilde{v}_0, \ \tilde{s}_1, \ \tilde{v}_1, \dots, \tilde{s}_N, \ \tilde{v}_N\right]^\top \in \mathbb{R}^n, \tag{11}$$

where $n = 2N+2$. From the linearized dynamics in (7)-(10), we have the dynamics of x as:

$$\dot{x}(t) = Ax(t) + \sum_{i=1}^{N} A_i x(t-\tau_{\mathrm{F}i}) + Bu(t-\tau_{\mathrm{U}}) + Dr(t), \tag{12}$$

where the model coefficients are

$$A = \begin{bmatrix} M_0 & & & \\ M_2 & M_1 & & \\ & \ddots & \ddots & \\ & & M_2 & M_1 \end{bmatrix}, B = \begin{bmatrix} b_0 \\ 0_{2\times1} \\ \vdots \\ 0_{2\times1} \end{bmatrix}, D = \begin{bmatrix} d_0 \\ 0_{2\times1} \\ \vdots \\ 0_{2\times1} \end{bmatrix},$$

$$M_0 = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, M_1 = \begin{bmatrix} 0 & -1 \\ 0 & a_{i2} \end{bmatrix}, M_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, b_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, d_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \tag{13}$$

and the matrix $A_i \in \mathbb{R}^{n\times n}$ with only two non-zero elements

$$(A_i)_{(2i+2,2i)} = a_{i3}, (A_i)_{(2i+2,2i+1)} = a_{i1}. \tag{14}$$

III. Safety-Critical Traffic Control

A. State Predictor for Delay Compensation

Assumption 1: For the time-delayed traffic system, the actuator delay is shorter than all state delays, i.e.,

$$\tau_{Fi} \geq \tau_U, \quad \forall i = 1, \ldots, N. \tag{15}$$

The assumption is consistent with real traffic system since human drivers usually have a longer reaction time than automated vehicles.

Assumption 2: The derivative of the disturbance $\dot{r}(t)$ has a constant upper bound $\bar{a} > 0$ and a constant lower bound $\underline{a} < 0$ such that $\forall t \geq 0$:

$$\underline{a} \leq \dot{r}(t) \leq \bar{a}. \tag{16}$$

The disturbance $r(t)$ is the HV's speed, so its derivative $\dot{r}(t)$ represents HV's acceleration. Given the physical constraints of vehicles, Assumption 2 thus holds in practice since $\underline{a}$ and $\bar{a}$ are the minimum and maximum acceleration.

Assumption 3: The time-delayed traffic system remains safe before initial $u(0)$ is actuated, i.e., for all $i = 0, \ldots, N$, we have $x(t) \in \mathcal{S}_i$, $\forall t \in [0, \tau_U]$.

For the mixed time-delayed traffic (12), by timing $\exp -\tau_U A$ on both sides, and integrating from t to $t+\tau_U$, its future state is given by historical state x, historical control input u, and future disturbance r as

$$\begin{aligned} x(t+\tau_U) &= e^{\tau_U \cdot A} x(t) \\ &+ \sum_{i=1}^{N} \int_{-\tau_{Fi}}^{\tau_U - \tau_{Fi}} e^{(\tau_U - \tau_{Fi} - \omega)\cdot A} A_i x(t+\omega)\, \mathrm{d}\omega \\ &+ \int_{-\tau_U}^{0} e^{-\omega \cdot A} B u(t+\omega)\, \mathrm{d}\omega \\ &+ \int_{0}^{\tau_U} e^{(\tau_U - \omega)\cdot A} D r(t+\omega)\, \mathrm{d}\omega. \end{aligned} \tag{17}$$

In traffic, the future disturbance value is unknown, so we predict the state using current $r(t)$ as:

$$\begin{aligned} \phi(t) &= e^{\tau_U \cdot A} x(t) \\ &+ \sum_{i=1}^{N} \int_{-\tau_{Fi}}^{\tau_U - \tau_{Fi}} e^{(\tau_U - \tau_{Fi} - \omega)\cdot A} A_i x(t+\omega)\, \mathrm{d}\omega \\ &+ \int_{-\tau_U}^{0} e^{-\omega \cdot A} B u(t+\omega)\, \mathrm{d}\omega \\ &+ \int_{0}^{\tau_U} e^{(\tau_U - \omega)\cdot A} D r(t)\, \mathrm{d}\omega. \end{aligned} \tag{18}$$

B. Safe Spacing Policy and Safety Constraint

We adopt the constant time headway (CTH) criterion as the safe spacing policy. For a vehicle i, given a safe time-headway $\psi_i > 0$, CTH restricts the safe spacing as:

$$s_i \geq \psi_i v_i. \tag{19}$$

For each vehicle, its safe set is:

$$\mathcal{S}_i = \left\{x \in \mathbb{R}^n : h_i(x) \geq 0\right\}, \tag{20}$$

with the safety function being:

$$h_i(x) = s_i - \psi_i v_i. \tag{21}$$

Theorem 1 (Safety Constraint for Traffic Systems With Simultaneous State and Actuator Delays): For the time-delayed traffic system, under Assumptions 1-3, for a Lipschitz continuous controller u, if

$$\begin{aligned} &L_f h_0(\phi(t), x(t-\tau_F+\tau_U)) + L_g h_0(\phi(t))u(t) \\ &\geq -\alpha_0(h_0(\phi(t))) + S_0(t), \end{aligned} \tag{22}$$

where the Lie derivatives are:

$$\begin{aligned} &L_f h_0(\phi(t), x(t-\tau_F+\tau_U)) \\ &= \nabla h_0(\phi(t)) \cdot \left(A\phi(t) + \sum_{i=1}^{N} A_i x(t-\tau_{Fi}+\tau_U)\right), \end{aligned} \tag{23}$$

$$L_g h_0(\phi(t)) = \nabla h_0(\phi(t)) B, \tag{24}$$

the safety margin is

$$\begin{aligned} S_0(t) &= \alpha_0(h_0(\phi(t))) - \alpha_0\left(h_0(x) + \underline{a}\tau_U^2/2\right) \\ &- r(t) - \underline{a}\tau_U, \end{aligned} \tag{25}$$

with $\phi(t)$ given by the predictor (18) and α_0 being an extended class $\mathcal{K}$ function, then CAV's safe set $\mathcal{S}_0$ (20) is forward invariant. If the controller u further satisfies

$$\begin{aligned} &L_f h_i^r(\phi(t), x(t-\tau_F+\tau_U)) + L_g h_i^r(\phi(t))u(t) \\ &\geq -\alpha_i\left(h_i^r(\phi(t))\right) + S_i(t), \end{aligned} \tag{26}$$

where function $h_i : \mathbb{R}^n \to \mathbb{R}$ is:

$$h_i^r = h_i - \lambda_i h_0, \tag{27}$$

with $\lambda_i > 0$ being a positive constant, $S_i(t)$ represents the safety margin:

$$\begin{aligned} S_i(t) &= \alpha_i\left(h_i^r(\phi(t))\right) - \alpha_i\left(h_i^r(\phi(t)) - \lambda_i \bar{a}\tau_U^2/2\right) \\ &+ \lambda_i r(t) + \lambda_i \tau_U \bar{a}, \end{aligned} \tag{28}$$

then safe set for FV-i $\mathcal{S}_i$ (20) is forward invariant.

Proof: We prove following three steps:

I. Bound the prediction error and derive robust safety function: The prediction error is

$$x(t+\tau_U) - \phi(t) = \int_0^{\tau_U} e^{(\tau_U-\omega)A} D(r(t+\omega) - r(t))\, \mathrm{d}\omega. \tag{29}$$

From (13), $e^{A\omega}D = D$ holds for any ω. For the predicted speed and gap of CAV, $v_{0,p}(t)$ and $s_{0,p}(t)$, we have bounds from Assumption 2:

$$v_0(t+\tau_U) - v_{0,p}(t) = 0, \tag{30}$$

$$\underline{a}\tau_{\rm U}^2/22 \le s_0(t+\tau_{\rm U}) - s_{0,p}(t) \le \bar{a}\tau_{\rm U}^2/2. \quad (31)$$

Therefore, to meet the CTH constraint in (19) for CAV, it is sufficient to have

$$s_{0,p}(t) + \underline{a}\tau_{\rm U}^2/2 \ge \psi_0 v_{0,p}(t). \quad (32)$$

Define

$$h_{0R}(\phi(t)) = s_{0,p}(t) - \psi_0 v_{0,p}(t) + \underline{a}\tau_{\rm U}^2/2, \quad (33)$$

as a robust safety function, then $h_{0R}(\phi(t)) \ge 0 \implies h_0(x(t+\tau_{\rm U})) \ge 0$.

II. Design safety constraints for the robust safety function: For the designed predictor (18), its dynamics are

$$\dot{\phi}(t) = A\phi(t) + \sum_{i=1} A_i x(t - \tau_{\rm F\mathit{i}} + \tau_{\rm U}) + Bu(t) + Dr(t) + D\tau_{\rm U}\dot{r}(t). \quad (34)$$

Since $\nabla h_0 = \nabla h_{0R}$, we have

$$\begin{aligned}\dot{h}_{0R}(\phi(t)) &= \nabla h_0(\phi(t))\dot{\phi}(t) \\ &= L_f h_0(\phi(t), x(t-\tau_{\rm F}+\tau_{\rm U})) + L_g h_0(\phi(t))u(t) \\ &\quad + \nabla h_0(\phi(t))Dr(t) + \nabla h_0(\phi(t))D\tau_{\rm U}\dot{r}(t). \quad (35)\end{aligned}$$

Since $\nabla h_0 D = 1$, so if (22) holds, then

$$\dot{h}_{0R}(\phi(t)) \ge -\alpha(h_{0R}), \quad (36)$$

which means $h_{0R}(\phi(t)) \ge 0$. So $\mathcal{S}_0$ is forward invariant.

For FV's safety function h_i, its relative degree with respect to control input u is $i+1$. Compared with CBF with relative degree one, the high-order CBF has a more complex formulation [1], [14]. We construct a reduced degree CBF for FV as (27). And by guaranteeing $h_i^r(x) \ge 0$ and $h_0(x) \ge 0$, we have $h_i(x) \ge 0$. Consider predicted speed and gap of FV-i, $v_{i,p}(t)$ and $s_{i,p}(t)$, we have from (29):

$$v_i(t+\tau_{\rm U}) = v_{i,p}(t), \quad (37)$$
$$s_i(t+\tau_{\rm U}) = s_{i,p}(t). \quad (38)$$

Therefore, consider a robust safety function:

$$h_{iR}^r = h_i^r - \lambda_i \bar{a}\tau_{\rm U}^2/2, \quad (39)$$

then $h_{iR}^r(\phi(t)) \ge 0 \implies h_i^r(x(t+\tau_{\rm U})) \ge 0$. By the definition of h_{iR}^r, $\nabla h_i^r = \nabla h_{iR}^r$, so

$$\begin{aligned}h_i^r(\phi(t)) &= \nabla h_i^r(\phi(t))\dot{\phi}(t) \\ &= L_f h_i^r(\phi(t), x(t-\tau_{\rm F}+\tau_{\rm U})) + L_g h_i^r(\phi(t))u(t) \\ &\quad + \nabla h_i^r(\phi(t))Dr(t) + \nabla h_i^r(\phi(t))D\tau_{\rm U}\dot{r}(t). \quad (40)\end{aligned}$$

Since $\nabla h_i^r D = -\lambda_i$, if (26) holds, then

$$\dot{h}_{iR}^r(\phi(t)) \ge -\alpha_i(h_{iR}^r(\phi(t))), \quad (41)$$

which means $h_{iR}^r(\phi(t)) \ge 0$ always holds. So we have the safe set $\mathcal{S}_i$ (20) is forward invariant. ■

Theorem 1 gives the safety constraints to make the vehicle chain safe. Given a nominal controller, a safety-critical controller is synthesized from a quadratic programming:

$$\begin{aligned}u = \underset{u\in\mathbb{R},\sigma_i\ge 0}{\operatorname{argmin}}\ & |u-u_0|^2 + \sum_{i=1}^{N} p_i\sigma_i^2 \\ \text{s.t.}\quad & L_f h_0(\phi(t), x(t-\tau_{\rm F}+\tau_{\rm U})) + L_g h_0(\phi(t))u(t) \\ & + \alpha_0(h_0(\phi(t))) - S_0(t) \ge 0, \\ & L_f h_1^r(\phi(t), x(t-\tau_{\rm F}+\tau_{\rm U})) + L_g h_1^r(\phi(t))u(t) \\ & + \alpha_1\big(h_1^r(\phi(t))\big) - S_1(t) + \sigma_1 \ge 0, \\ & \dots \\ & L_f h_N^r(\phi(t), x(t-\tau_{\rm F}+\tau_{\rm U})) + L_g h_N^r(\phi(t))u(t) \\ & + \alpha_N\big(h_N^r(\phi(t))\big) - S_N(t) + \sigma_N \ge 0, \quad (42)\end{aligned}$$

where $\phi(t)$ is given by the predictor (18), σ_i are slack variables, and $p_i > 0$ are penalty coefficients. We note that the first constraint of this QP is the same as the CAV safety (22) in Theorem 1. The following N constraints are a relaxed version of the following vehicles' safety constraint (26). To ensure feasibility of the QP, CAV safety is prioritized over FV safety by introducing slack variables into FV safety constraints. Such an approach is justified by the following two aspects. The controller is actuated to the CAV and so shoulders primary responsibility for CAV safety. If the CAV has a collision, FVs are also affected, but not vice versa.

IV. Numerical Simulation

A. Simulation Setting

We identify two safety-critical scenarios.

- The HV suddenly decelerates. In traffic This may happen when because of a sudden cut-in. We set the HV to decelerate with a deceleration a_H and duration t_H.
- One FV suddenly accelerates. This may happen when the driver has fatigue deriving or wrong operations. We set FV's accidental acceleration as a_F with duration t_F.

We call the system *safe* if the safety measure h (21) remains positive (or zero) during the control process, and *unsafe* if h becomes negative. We consider a vehicle chain of one HV, followed by the controlled CAV and $N = 2$ FVs. We adopt the car-following model for the FV:

$$F_i(s_i, v_i, v_{i-1}) = a(V(s_i(t-\tau_{Fi})) - v_i(t)) + b(v_{i-1}(t-\tau_{Fi}) - v_i(t)), \quad (43)$$

where $V(s)$ presents the desired speed. We take $V(s)$ the same as [18]. We take the state delay as $\tau_{\rm F1} = \tau_{\rm F2} = 0.5$ s and the actuator delay as $\tau_{\rm U} = 0.2$ s. For the CBF-related parameter in the designed safety-critical controller (42), we take $\alpha_i(x) = 10x$, $\eta_i = 0.9$, and $p_i = 100$. We take the time headway in the CTH (19) as $\psi_i = 0.5$ s. We take $v^* = 20$ m/s as the equilibrium speed, which gives the equilibrium gap from $F_i(s_i^*, v^*, v^*) = 0$ as $s_i^* = 20$ m. We take the nominal controller in a feedback law [12]:

$$u_0(t) = K\phi(t) + \alpha_3 r(t), \quad (44)$$

with the feedback gain $K = [\alpha_1, -\alpha_2, -5, 0.5, -1, 0.2]$.

B. Safety Guarantee by Safety-Critical Controller

In Fig. 2, we give the simulation results of the nominal controller (44) (the first row in Fig. 2) and the safety-critical controller (42) (the second row in Fig. 2) under Scenario 1 when the head vehicle suddenly accelerates. We take the

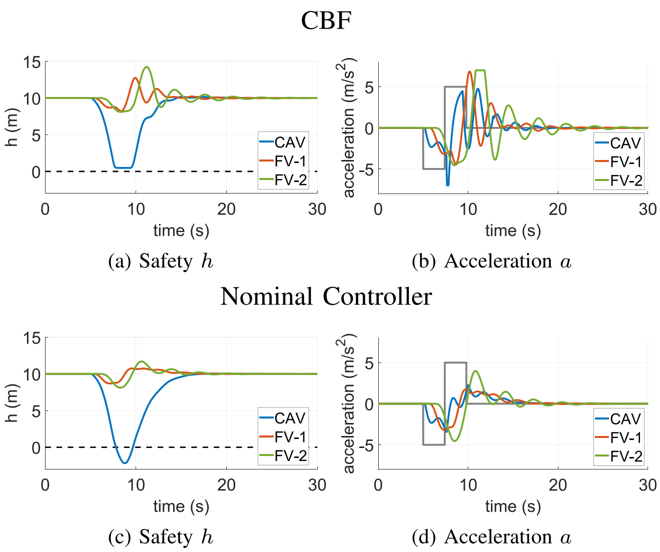

Fig. 2. Trajectory when the head vehicle suddenly decelerates.

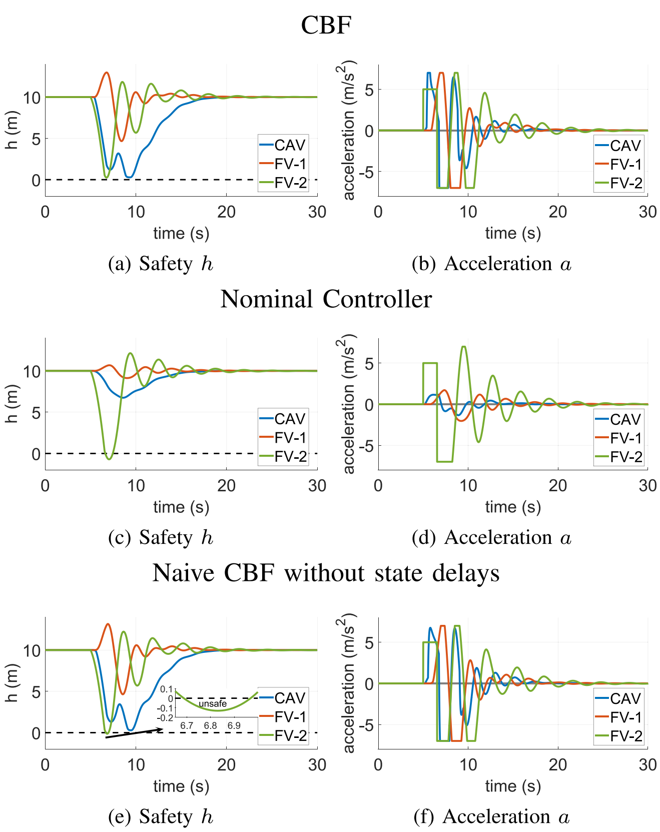

Fig. 3. Trajectory when the FV-2 suddenly accelerates.

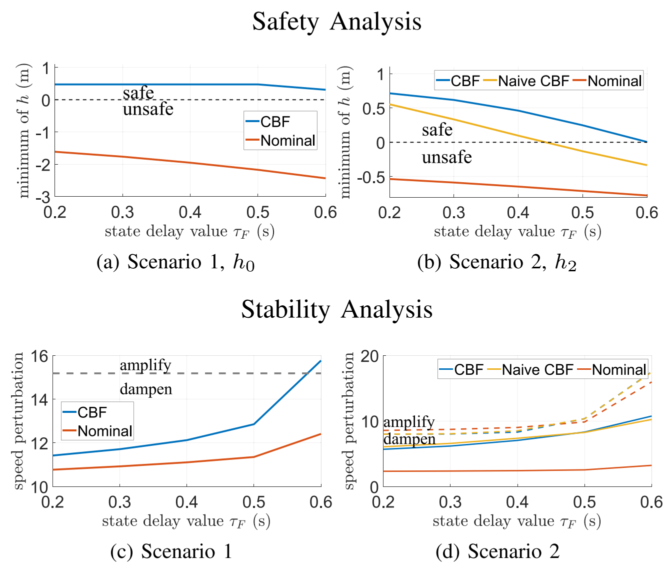

Fig. 4. Stability and safety analysis.

accidental deceleration as $a_H = -5$ m/s with $t_H = 2.4$ s. When HV suddenly decelerates, the nominal controller tries to stabilize the traffic and has a smaller deceleration of the CAV, which causes a collision between CAV and HV. The safety-critical controller has a large acceleration to ensure safety. We note that in this scenario, the naive CBF without considering the state delays [18] have the same trajectories as (42) that considers the state delays. This is because that the CAV has no delays in its state s_0 and v_0 as in (3)-(4), and state delay comes from the following vehicles as in (1)-(2). In this scenario, the nominal controller violated the safety of the CAV, but not the safety of the following vehicles. So CBFs with and without state delays have the same trajectories.

In Fig. 3, we give the simulation result of the safety-critical controller (42) (the first row), the nominal controller (44) (the second row) and the safety-critical controllers without state delay [18] (the third row) in Scenario 2. We set the FV-2 to accidentally accelerate with acceleration as $a_F = 5$ m/s and duration $t_F = 1.5$ s. In this scenario, when the following FV-2 suddenly accelerates, to guarantee safety, the CAV also begins to accelerate as in Fig. 3(d). This guides FV-1 to accelerate and thus increases the gap between FV-1 and FV-2 and improves the safety of FV-2. If ignoring the state delays, the naive CBF [18] fails to maintain safety of the system as Fig. 3(i) shows.

C. CBF's Effect on Stability and Safety

We analyze how the stability and safety of the system are affected by different controllers. We re-run simulations with different state delay values τ_{F} ranging from 0.2 s (τ_{U}) to 0.6 s, but with other parameters the same as above. Note that we require $\tau_{\text{F}} \geq \tau_{\text{U}}$ in Assumption 1.

We evaluate the system safety under different controllers in Fig. 4(a) and Fig. 4(b) via the minimum value of the safety measure h during the control process. A higher h means the system is safer. With a negative h, the system is considered unsafe and is at a high risk of collisions.

In Fig. 4(a) for the first scenario, the safety risk is at the CAV, so we plot the minimum value of h_0 in Fig. 4(a). We see that the designed safety-critical controllers guarantee safety for the system for all considered state delay values, i.e., h remains positive.

For the second scenario, the safety risk is at FV-2, so we give the minimum value of h_2 in Fig. 4(b). Under all considered state delay values, the designed safety-critical controller has a higher h value and thus improves the system safety. We also note that when $\tau_F > 0.5$, the naive CBF [18] renders $h_2 < 0$, which makes the system unsafe. This shows the necessity of re-designing the safety constraints when there are state delays in the system.

To evaluate the traffic stability, we compare the speed perturbation of the accident vehicle, I_0, and the average speed perturbation of other vehicles I in the L_2-Norm. If $I < I_0$, the perturbations in the vehicle chain are dampened by the CAV, and the traffic becomes smoother. We have I_0 and I for the two scenarios as follows:

- For the first scenario when the head vehicle accelerates, the head vehicle is the accident vehicle, so we have I_0 as:

$$I_0 = \sqrt{\int_0^T (v_{-1}(t) - v^*)^2 \, dt}, \tag{45}$$

with $T = 30$ s being the total simulation time. The CAV (indexed as 0) and the two following vehicles (indexed as 1 and 2) are all affected by this accident, so we have I as:

$$I = \frac{\sum_{i=0}^{2} \sqrt{\int_0^T (v_i(t) - v^*)^2 \, dt}}{3}. \tag{46}$$

Note that in this case, the motion of head vehicle is not affected by the CAV, so I_0 remains the same for different controllers.

- In the second scenario, the FV-2 suddenly accelerates. So I_0 is:

$$I_0 = \sqrt{\int_0^T (v_2(t) - v^*)^2 \, dt}. \tag{47}$$

But the head vehicle remains at the equilibrium speed and is not affected, since it only adapts its speed based on downstream traffic, not the following vehicles. So we have I and I_0 as:

$$I = \frac{\sum_{i=0}^{1} \sqrt{\int_0^T (v_i(t) - v^*)^2 \, dt}}{2}. \tag{48}$$

Note that in this case, since the deceleration process of FV-2 is also affected by the controlled CAV, I_0 is different with different controllers.

Fig. 4(c) and Fig. 4(d) analyze the stability of the system under the two scenarios. The solid and dashed curves give I and I_0 respectively. For Scenario 1, we note that the CBF has a higher I than the nominal controller. This is because the CAV has a larger deceleration to avoid collisions as shown in Fig. 2. But within a large range of state values, approximately $\tau_F < 0.58$ s, the CBF still has $I < I_0$, i.e., the perturbations in the leading vehicle is not amplified. For Scenario 2, as Fig. 4(d) shows, for all $\tau_F \leq 0.6$ s, we have $I < I_0$ even with the safety-critical controller. To summarize the four subfigures in Fig. 4, the designed safety-critical controller (42) achieves safe stabilizing for the traffic system.

V. Conclusion

In this letter, we consider safe stabilizing control of time-delayed traffic systems with both actuator and state delays. A predictor is designed to compensate for the actuator delay. We develop novel CBF safety constraints that address both state delays, actuator delay and the prediction error caused by disturbances from the head vehicle. For time-delayed traffic system, safe spacing of both CAV and the following vehicles are considered. A safety-critical controller is synthesized to impart formal safety guarantee for the time-delayed system. Future work includes coordinating multiple CAVs and designing lateral controllers.

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